ON THE DISTRIBUTION OF THE "STUDENT" RATIO FOR SMALL SAMPLES FROM CERTAIN NON-NORMAL POPULATIONS[1]

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Much of interest in the theory and practice of statistical methods has been developed around the distribution function,

$$(1) \qquad \frac{\Gamma(N/2)}{\pi^{\frac{1}{2}} \Gamma\left(\dfrac{N-1}{2}\right)(1+z^2)^{N/2}}$$

of the "Student" ratio, $z = \dfrac{\bar{x} - m}{s}$, where $\bar{x}$ denotes the mean, s the standard deviation of a sample of N items, say $x_1, x_2, \cdots, x_N$, taken at random from a normally distributed parent population of mean, m.

The investigations of certain non-normal parent distributions by Shewhart and Winters [1], Rider [2], E. S. Pearson [3], M. S. Bartlett [4], and R. C. Geary [5] indicate that applications of the "Student" theory give more satisfactory results than the classical theory for a considerable variety of non-normal parent distributions, but some of these investigators find that the theory fails in certain cases to describe the facts to an extent that suggests further experimental sampling investigations along this line whenever suitable data are available. Others infer that a completely satisfactory analysis of the position of the "Student" z-test will be possible only if the theoretical distribution of z in samples from the non-normal distribution in question becomes known. Several of the above named statisticians have attributed the failures of the distribution (1) to describe their data, in large part, to the correlation between $x = \bar{x} - m$ and s. For this reason, there is considerable interest in the degree of correlation between $x = \bar{x} - m$ and s, and especially in the nature of the regression of s or of s^2 on x.

The present paper gives an analysis of data obtained by experimental sampling from two non-normal distributions whose sources we shall now describe. The parent distributions with which the paper is concerned are theoretical distributions resulting from certain urn schemata devised [6] by the writer some years ago.

In 1925, Leone E. Chesire, in an unpublished thesis for the degree, Master of Science, at the University of Iowa, obtained data by experimental sampling, that seem to be appropriate material for a study of the correlation of mean and standard deviation for small samples from certain non-normal distributions.

One of the original bivariate parent populations, whose marginal totals we are

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