NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

THE DISTRIBUTION OF "STUDENT'S" RATIO FOR SAMPLES OF TWO ITEMS DRAWN FROM NON-NORMAL UNIVERSES

By Jack Laderman

The fundamental assumption in the derivation of "Student's" distribution[1]

$$f(t)\,dt = \frac{\Gamma\left(\frac{n}{2}\right)}{\sqrt{\pi(n-1)}\;\Gamma\left(\frac{n-1}{2}\right)\left(1+\frac{t^2}{n-1}\right)^{\frac{1}{2}n}}\,dt$$

is that the universe sampled is normal. When the universe sampled is non-normal and n is small, the distribution of t differs considerably from "student's" distribution. In 1929, Rider[2] derived the distribution of t for samples of two drawn from the rectangular distribution

$$f(x)\,dx = \begin{cases} 1\,dx & \text{for } 0 \leq x \leq 1 \\ 0 & \text{elsewhere.} \end{cases}$$

In this paper, the formal expression for the distribution of t will be derived for samples of two drawn from any population having a continuous frequency function. A geometrical method similar to that employed by Rider will be used.

Let the universe sampled have the frequency function, $f(x)$, with zero mean, and let $f(x)$ be greater than zero from $x = a$ to $x = b$ and equal to zero elsewhere. Suppose the two observations are x_1 and x_2.

Then
$$\bar{x} = \frac{x_1 + x_2}{2}$$

and
$$t = \frac{\sqrt{n}\,\bar{x}}{\sqrt{\frac{\Sigma(x-\bar{x})^2}{n-1}}} = \frac{\sqrt{2}\,\bar{x}}{\sqrt{(x_1-\bar{x})^2 + (x_2-\bar{x})^2}}.$$

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[1] "Student", "The Probable Error of a Mean" *Biometrika*, Vol. VI (1908), pp. 1–25.

[2] Paul R. Rider, "On the Distribution of the Ratio of the Mean to Standard Deviation in Small Samples from Non-Normal Universes", *Biometrika*, Vol. XXI (1929), pp. 124–143.