ON TESTING THE HYPOTHESIS THAT TWO SAMPLES HAVE BEEN DRAWN FROM A COMMON NORMAL POPULATION

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1. **Introduction.** This paper is devoted to the problem of testing the hypothesis that two samples of 2, 3 and 4 variables, and of equal size, have been drawn from a common unspecified normal population. It is, in a certain sense, a continuation of J. W. Fertig's papers [1, 2] which were devoted to the problem of testing the hypothesis that one or more samples of n variables have been drawn from a completely or partially specified normal population.

For the sake of application to biological research, it is important to have means of determining whether two samples may have come from a common population even if this population is unknown. Moreover, it is often imperative to test two samples with respect to all their variables simultaneously. Much valuable information may be lost if the variables are tested individually. One has to consider not only the fact that two samples which differ almost significantly from each other in each variable separately might be significantly different if the probabilities would be combined, but one has to take account of the possible correlations between the variables which are completely disregarded if the tests are applied to each variable separately. It is not difficult to imagine two samples of two variables with identical means and variances and significantly different correlation coefficients.

J. Neyman and E. S. Pearson [3] have investigated the problem of testing statistical hypotheses in general. They have developed the method of likelihood ratios. It is beyond the scope of the present paper to give an account of this theory; we have to restrict ourselves to statements concerning the fundamental concepts we are going to apply to our specific problem.

A sample with one variable and of size N can be regarded as a point in an N-dimensional space. The acceptance or rejection of a hypothesis concerning this sample will depend on whether or not the point representing the sample is contained in certain critical regions determined by the hypothesis and by the statistical criterion that is to be applied. The choice of the critical regions is of fundamental importance; its implications have been thoroughly discussed by Neyman and Pearson. These authors found a useful criterion for testing the hypothesis that a sample was drawn from a specified member of an admissible set of populations by introducing the ratio of the likelihood that the sample was drawn from the specified population to the maximum value of the likelihood for all populations in the admissible set (Cf. §2). This ratio λ can vary between

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