A LEAST SQUARES ACCUMULATION THEOREM

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The following simple least squares theorem does not seem to have been mentioned in the literature, and has at least one practical application.

If $A^*(x)$ and $B^*(x)$ are polynomials of the same degree which are least squares representations of the functions $A(x)$ and $B(x)$ respectively, for the values $x_1, x_2, x_3, \cdots, x_p$, then

$$\sum_{t=1}^{p} A^*(x_t)B(x_t) = \sum_{t=1}^{p} A(x_t)B^*(x_t) = \sum_{t=1}^{p} A^*(x_t)B^*(x_t). \tag{1}$$

To prove the theorem let

$$A^*(x) = \sum_{i=0}^{m} a_i x^i \tag{2}$$

and

$$B^*(x) = \sum_{j=0}^{n} b_j x^j. \tag{3}$$

Then the normal equations for the determination of a_i and b_j are

$$\sum_{i=0}^{m} a_i s_{i+k} = \sum_{t=1}^{p} x_t^k A(x_t), \qquad k = 0, 1, 2, \cdots, m, \tag{4}$$

and

$$\sum_{j=0}^{n} b_j s_{j+h} = \sum_{t=1}^{p} x_t^h B(x_t), \qquad h = 0, 1, 2, \cdots, n, \tag{5}$$

where $s_r = \sum_{t=1}^{p} x_t^r$. Hence, by (2) and (5)

$$\begin{aligned} \sum_{t=1}^{p} A^*(x_t)B(x_t) &= \sum_{t=1}^{p} \left[\sum_{i=0}^{m} a_i x_t^i\right] B(x_t) \\ &= \sum_{i=0}^{m} a_i \sum_{t=1}^{p} x_t^i B(x_t) \\ &= \sum_{i=0}^{m} \sum_{j=0}^{n} a_i b_j s_{j+i} \quad \text{if} \quad n \geqq m, \\ &= \sum_{t=1}^{p} A^*(x_t)B^*(x_t) \quad \text{if} \quad n \geqq m. \end{aligned} \tag{6}$$

Similarly it can be shown that

$$\sum_{t=1}^{p} A(x_t)B^*(x_t) = \sum_{t=1}^{p} A^*(x_t)B^*(x_t) \quad \text{if} \quad m \geqq n. \tag{7}$$