COMPARISON OF PEARSONIAN APPROXIMATIONS WITH EXACT SAMPLING DISTRIBUTIONS OF MEANS AND VARIANCES IN SAMPLES FROM POPULATIONS COMPOSED OF THE SUMS OF NORMAL POPULATIONS

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1. Introduction. Biological and sociological data are often "non-homogeneous" and of such a nature as not to be easily separated into components. Non-homogeneous populations have been discussed by Karl Pearson, Charlier, and others. Non-normal material has been discussed by many writers. See for example, A. E. R. Church [1] and J. M. LeRoux [2] for a discussion of moments of the distributions of the means and variances for samples from non-normal material.

In a previous paper [3] the author has given the distributions of the means and standard deviations of samples from certain non-homogeneous populations. The purpose of the present paper is to extend the results given in [3] and to compare the moment approach of the Pearsonian school with the true distributions.

2. Moments of the distribution of means of samples of n from a non-homogeneous population. Consider a population with distribution

$$f(x) = \frac{1}{(1+k)\sqrt{2\pi}}\left[e^{-\frac{1}{2}x^2} + \frac{k}{\sigma}e^{-\frac{1}{2\sigma^2}(x-m)^2}\right]. \tag{2.1}$$

The first four moments of (2.1) about $x = 0$ are

$$\begin{aligned} v_1' &= \frac{km}{1+k} \\ v_2' &= \frac{1}{1+k}[1 + k(\sigma^2 + m^2)] \\ v_3' &= \frac{km}{1+k}[3\sigma^2 + m^2] \\ v_4' &= \frac{1}{1+k}[3 + k(3\sigma^4 + 6m^2\sigma^2 + m^4)]. \end{aligned} \tag{2.2}$$

The means of samples of n drawn at random from (2.1) are distributed according to

$$\frac{n}{\sqrt{2\pi}(1+k)^n}\left[\sum_{s=0}^{n}\binom{n}{s}\frac{k^s}{\sqrt{s\sigma^2+n-s}}\exp\left\{-\frac{n^2\left(x-\frac{s}{n}m\right)^2}{s\sigma^2+n-s}\right\}\right]. \tag{2.3}$$