REDUCTION OF A CERTAIN CLASS OF COMPOSITE STATISTICAL HYPOTHESES

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1. Introduction. A situation frequently met in sampling theory is the following: x has distribution $f(x, \theta)$, where θ is an unknown parameter, and for samples $(x_1, \cdots, x_n)$ there exists in the sample space E_n a family of $(n - 1)$-dimensional manifolds upon each of which the distribution is independent of θ; in addition there is a residual one-dimensional manifold available for estimating θ. For example, suppose there exists a sufficient statistic T for θ, then on the manifolds $T = T_0$ there is defined an induced distribution which is independent of the parameter.

A similar situation is observed when θ is a "location" or "scale" parameter. Let x have the distribution $f(x - a)$ for some a, then the set $(x_2 - x_1, x_3 - x_1, \cdots, x_n - x_1)$, or any equivalent set, such as $(x_2 - \bar{x}, \cdots, x_n - \bar{x})$, have a joint distribution independent of a, and there is a residual distribution corresponding to each particular configuration $(x_2 - x_1, \cdots, x_n - x_1)$. Fisher [1] and Pitman [5] have examined the residual distributions in connection with the problem of estimating scale and location parameters. In this paper we shall be concerned primarily, not with the residual distribution, but with the remainder of the sample information, corresponding to the $(n - 1)$-dimensional distribution which is independent of the parameter. It is found, in a rather broad class of distributions, that the part of the sample *not* used for estimation *determines, except for the parameter value, the original functional form of the distribution of x.*

This paper is devoted mainly to a study of particular classes of distributions having the property mentioned above. We consider also the theoretical application of this property to certain types of *composite* hypotheses which may be reduced thereby to equivalent *simple* hypotheses.[1] The principal results of this nature may be summed up as follows: If x has distribution of the form $f(x, \theta)$, where θ is either a location or scale parameter, or a vector denoting both, then there exists, in samples $(x_1, \cdots, x_n)$ a set of functions $y_i(x_1, \cdots, x_n)$, $i = 1, 2, \cdots, p$, $p < n$, having joint distribution $D(y_1, \cdots, y_p)$ independent of θ, and such that the converse statement holds, namely, *if* $\{y_i\}$ *have the distribution* $D(y_1, \cdots, y_p)$, *then* x *has, for some* θ, *a distribution of the form* $f(x, \theta)$. There is a corresponding statement when x has a distribution of the form $f(x - \Sigma a_i u_i)$, where the $\{a_i\}$ are parameters, and the $\{u_i\}$ are regression variables.

[1] We use the terms simple and composite hypotheses in the sense of Neyman and Pearson [2].