A SYMMETRIC METHOD OF OBTAINING UNBIASED ESTIMATES AND EXPECTED VALUES

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The problem of finding the relationship between moment functions of a sample and moment functions of the population from which the sample was obtained has, of necessity, received much attention. The problem has two parts: first, to find the expected value of a given sample moment function; second, to find the estimate of a given population moment function. Thus, if m_i represent the ith central moment of a sample and μ_i represent the ith central moment of the population, the first part of the problem requires that we find the mean value of m_i for all possible samples of a given size and express it in term of the μ_i's. The second part requires that we find a function of the m_i's such that the mean value, taken for all possible samples of a given size, be a given μ_i. For the case $i = 4$ we have the well known results:

$$E[m_4] = \frac{(n-1)(n^2-3n+3)}{n^3}\mu_4 + \frac{3(n-1)(2n-3)}{n^3}\mu_2^2,$$

$$E^{-1}[\mu_4] = \frac{n^2(n^2-2n+3)}{n^{(4)}} m_4 - \frac{3n^2(2n-3)}{n^{(4)}} m_2^2.$$

These results are based on the assumption of an infinite population. In spite of the inverse relationship existing between estimates and expected value, the expressions above show no simple relationship. This lack of simplicity of relationship between estimate and expected value is directly traceable to the fact that such results are usually obtained for infinite populations. When results are obtained for finite populations a symmetry is found to exist which reduces to a single problem the two parts stated above. Since this should be evident to anyone upon reflection, the main purpose of the present paper may be considered as that of indicating one method of demonstrating the result stated above as well as showing relationship of this method to material appearing in previously published papers.

Consider a finite population consisting of N items $x_1 \cdots x_N$ and samples of n items taken from that population, the sampling being done without replacement. We shall utilize the power product notation of P. S. Dwyer [1; p. 13]

$$(1) \qquad (q_1 \cdots q_r) = \sum_{i_1 \neq i_2 \neq \cdots \neq i_r}^{n} x_{i_1}^{q_1} x_{i_2}^{q_2} \cdots x_{i_r}^{q_r}$$