A CONCISE ANALYSIS OF CERTAIN ALGEBRAIC FORMS

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Many of the statistics in common use are functions of homogeneous algebraic forms in the items of the sample. Among such statistics are the mean, a linear form; the variance, a quadratic form; and the product moment, a bilinear form. With the extension of the science, the mathematical statistician is faced with the study of more complex statistics and the associated algebraic forms and matrices. The purpose of this paper is to set forth concise and efficient notations and methods which may be used in such analysis.

We shall borrow the essential features of our notation from differential geometry and tensor analysis. The Kroneker delta is defined as,

$$\delta_i^j = 1, \qquad i = j,$$
$$= 0, \qquad i \neq j.$$

The summation convention provides that summation be performed with respect to any index appearing twice in the same term. Thus,

$$x_i y^i = x_1 y^1 + x_2 y^2 + \cdots .$$

To extend the use of the summation convention, we shall frequently place indices on the numeral, 1. Thus,

$$1^i x_i = 1^1 x_1 + 1^2 x_2 + \cdots = x_1 + x_2 + \cdots .$$

Symmetry in the calculations is more striking if the pair of summation indices appears, one as a superscript, the other as a subscript. Therefore we allow the shifting of an index from the one position to the other at will. Thus,

$$x_i \equiv x^i.$$

Where no confusion will arise, indices may be placed outside of parentheses.

$$\left(\delta\delta - \frac{1}{a}\frac{1}{b}\right)_{ik}^{jl} = \delta_i^j \delta_k^l - \left(\frac{1_i^j}{a}\right)\left(\frac{1_k^l}{b}\right).$$

The standard notations for averages will be used.

$$\bar{x}_{i\cdot} = \left(\frac{1}{b}\right)^j x_{ij} = \left(\frac{1}{b}\right)\Sigma_j x_{ij} \tag{1}$$

$$\bar{x} = \left(\frac{1}{N}\right)\Sigma x_{ijk\ldots} . \tag{2}$$