where F_2 is the upper and F_1 the lower critical value of the analysis of variance distribution with $p_u - 1$ and $N - \sum_{u=1}^{r} p_u + r - 1$ degrees of freedom. In case of a single criterion of classification the confidence limits (8) are identical with those given in my previous paper.

THE FREQUENCY DISTRIBUTION OF A GENERAL MATCHING PROBLEM

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1. Introduction. This paper considers the matching of two decks of cards of arbitrary composition, and the complete frequency distribution of correct matchings is obtained, thus solving a problem proposed by Stevens.[1] It is also shown that the results can be interpreted in terms of a contingency table.

Generalizing a problem considered by Greenwood,[2] let us consider the matching of two decks of cards consisting of t distinct kinds, all the cards of each kind being identical. The first or "call" deck will be composed of i_1 cards of the first kind, i_2 of the second, etc., such that

$$i_1 + i_2 + i_3 + \cdots + i_t = n;$$

and the second or "target" deck will contain j_1 cards of the first kind, j_2 of the second, etc., such that

$$j_1 + j_2 + \cdots + j_t = n.$$

Any of the i's or j's may be zero. It is desired to calculate, for a given arrangement of the "call" deck, the number of possible arrangements of the "target" deck which will produce exactly r matchings between them ($r = 0, 1, 2, \cdots, n$). It is clear that these frequencies are independent of the arrangement of the call deck. For convenience the call deck may be thought of as arranged so that all the cards of the first kind come first, followed by all those of the second kind, and so on.

2. Formulae for the frequencies. Let us consider the number of arrangements of the target deck which will match the cards in the k_1th, k_2th, $\cdots$, k_sth positions in the call deck, regardless of whether or not matchings occur elsewhere. Let the cards in these s positions in the call deck consist of c_1 of the first kind, c_2 of the second, etc. Then:

$$c_1 + c_2 + \cdots + c_t = s.$$

The number of such arrangements of the target deck is

$$\frac{(n-s)!}{\prod_{h=1}^{t} (j_h - c_h)!}. \tag{1}$$

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[1] W. L. Stevens, *Annals of Eugenics*, Vol. 8 (1937), pp. 238–244.
[2] J. A. Greenwood, *Annals of Math. Stat.*, Vol. 9 (1938), pp. 56–59.

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