ON MECHANICAL TABULATION OF POLYNOMIALS

By J. C. McPherson

International Business Machines Corporation

1. Introduction. The purpose of this paper is to show how automatic accounting machines, which have been used previously in evaluating such quantities as Σx^n and $\Sigma x^{n-1}y$, may be used in the preparation of mathematical tables of integral powers, of polynomials, and of functions which can be approximated by polynomials. These tables may be prepared for any desired intervals of the argument such as 1, $\frac{1}{10}$, $\frac{1}{100}$, $\frac{1}{2}$, $\frac{1}{3}$, etc.

The method is an adaptation of the general theory of "cumulative" or "progressive" totals which has proved useful in computing moments and product moments both with and without accounting machines. The reader unfamiliar with the mathematical method and its machine applications might refer to such presentations as those of Hardy [1], Mendenhall and Warren [2, 3], Razram and Wagner [4], Brandt [5], and Dwyer [6, 7]. The main feature of the method is the computation of summed products or of summed powers by means of successive cumulated additions. It is shown in this paper how it is possible to use this same process in constructing tables of powers and tables of polynomials.

2. The Cumulative Formulas. If the numbers F_x are defined and finite for $x = 1, 2, 3, \cdots, (a - 1), a$, and if these values of F_x are cumulated for $x = a$, $x = a - 1$, etc., then the value in the row headed by $x = 1$ can be written as ${}^1T_1$. If these cumulations are cumulated successively with the superscript indicating the order of the cumulation and the subscript indicating the value of x which heads the row, then

$$ {}^2T_1 = \Sigma x F_x, \qquad {}^3T_1 = \Sigma \frac{(x+1)x}{2!} F_x, \qquad {}^3T_2 = \Sigma \frac{x(x-1)}{2!} F_x, $$

$$ {}^4T_1 = \Sigma \frac{(x+2)(x+1)x}{3!} F_x $$

and in general for $i < j$,

$$ {}^jT_i = \Sigma \frac{[x + j - (i+1)]^{(j-1)}}{(j-1)!} F_x. \tag{1} $$

Formula (1) is basic to much of the previous work involving cumulative totals. Various authors have studied such important special cases as (A) where F_x equals the frequency function f_x, (B) where $F_x = xf_x$, and (C) where F_x equals the sum of all the values of y having the same x value. These special cases have been found very useful in computing moments and product moments.