ON CERTAIN LIKELIHOOD-RATIO TESTS ASSOCIATED WITH THE EXPONENTIAL DISTRIBUTION

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Various likelihood-ratio tests and their distributions in samples from a population having the elementary probability law $\frac{1}{\sigma} e^{-(x-B)/\sigma}$, $B \leq x \leq \infty$, have been studied by Neyman and Pearson [1] and Sukhatme [2]. In this note the power functions and the question of bias of several likelihood-ratio tests will be investigated. The exponential distribution appears to be appropriate for dealing with problems involving the intervals of time between events which tend to be random, as for example the interval between consecutive telephone calls, or the interval between consecutive accidents to the same worker.

To test the hypothesis H' that the location parameter B is equal to some fixed value, it being assumed that the scale parameter σ is known, we can for simplicity take the set Ω of admissible populations from which the sample might have been drawn to be $\{-\infty < B < +\infty, \sigma = 1\}$, while the subset ω from which the sample must come when the hypothesis is true is $\{B = 0, \sigma = 1\}$. Then the likelihood-ratio λ_1 for testing this hypothesis is

$$\lambda_1 = \frac{P(\omega \text{ max.})}{P(\Omega \text{ max.})} = \frac{e^{-\sum_{i=1}^{n} x_i}}{e^{-\sum_{i=1}^{n} (x_i - x_1)}} = e^{-nx_1},$$

where x_1 is the smallest observation in a random sample of n. The region of acceptance of this hypothesis consists of all points in sample space for which

$$\lambda_{1\epsilon} \leq \lambda_1 \leq 1,$$

where $\lambda_{1\epsilon}$ is chosen so that $\int_{\lambda_{1\epsilon}}^{1} g_1(\lambda_1)\, d\lambda_1 = 1 - \alpha$, α being the level of significance used and $g\ (\lambda_1)\, d\lambda_1$ being the distribution of λ_1 when B is really equal to zero. The region $\lambda_{1\epsilon} \leq \lambda_1 \leq 1$ is equivalent to the region in the sample space for which

$$0 \leq x_1 \leq k_1\,;\, k_1 = -\frac{\log \lambda_{1\epsilon}}{n}.$$

For any value of B the distribution of x_1 is known [3] to be

$$\phi_1(x_1)\, dx_1 = ne^{-n(x_1 - B)}\, dx_1\,.$$

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