NOTE ON A METHOD OF SAMPLING

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Olds[1] has considered the following problem: *Given a lot of size $m = s + r$ containing s items of a specified kind. Items are drawn without replacement until j of the s items have been drawn. The problem is to determine the probability law of n, the number of drawings which have to be made.* In the present note, we shall consider a certain limiting form for the probability function of n and make some remarks concerning repeated sampling of this type.

If n is the size of a drawing $j \leq n \leq r + j$ its probability law $P(n)$ is given by:

$$P(n) = \frac{C_{r,n-j}C_{s,j}}{C_{m,n}} \cdot \frac{j}{n} = \frac{\Gamma(s+1)}{\Gamma(j)\Gamma(s-j+1)} C_{r,n-j} \int_0^1 x^{n-1}(1-x)^{m-n}\, dx.$$

The characteristic function of n is

$$\varphi(t, n) = \sum_{n=j}^{r+j} P(n)e^{nt} =$$

$$\frac{\Gamma(s+1)}{\Gamma(j)\Gamma(s-j+1)} \cdot e^{jt} \int_0^1 x^{j-1}(1-x)^{s-j}(1-x+xe^t)^r\, dx.$$

Differentiating we find

$$\frac{\varphi'(t, n)}{\varphi(t, n)} = j + re^t \frac{\int_0^1 x^j(1-x)^{s-j}(1-x+xe^t)^{r-1}\, dx}{\int_0^1 x^{j-1}(1-x)^{s-j}(1-x+xe^t)^r\, dx} \tag{1}$$

and hence

$$m_1(n) = \sum_{n=j}^{r+j} P(n)n = [\varphi'(t, n)]_{t=0} = j \cdot \frac{m+1}{s+1}.$$

For the calculation of moments about the mean we take

$$\varphi(t, n - m_1) = e^{-m_1 t}\varphi(t, n), \tag{2}$$

from which we obtain

$$[\varphi^{(k)}(t, n - m_1)]_{t=0} = \sum_{n=j}^{r+j} P(n)(n - m_1)^k = \mu_k(n).$$

In particular, $\mu_2 = \dfrac{rj(m+1)(s+1-j)}{(s+1)^2(s+2)}$. The values of $m_1(n)$ and $\mu_2(n)$ have already been given by Olds using another method. Putting $\dfrac{rj}{s+1} = \beta$, we have

[1] E. G. Olds, *Annals of Math. Stat.*, Vol. 11 (1940), p. 355.