

SOLUTION OF A MATHEMATICAL PROBLEM CONNECTED WITH THE THEORY OF HEREDITY¹

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Translator's Note: Although a French resumé of the article here translated appeared in *Comptes Rendus*,² it is so condensed due to space restrictions that in reporting on Bernstein's work for the Statistical Seminar at the University of California, it became necessary to refer to the original Russian paper. Because of the obvious language difficulty together with the extreme rareness³ of the Ukrainian publication in this country, and because of the current interest in the application of statistical theories to genetics, it seemed advisable to make this important article available to a much larger class of readers.

It is regretted that, due to the present conditions, it was impracticable to obtain the author's comments on this translation, and it is hoped that the slight changes and additions inserted, to clarify some of the more difficult passages, would have met with his approval.

1. Let us consider N classes of individuals which possess the property that the cross of any two of these individuals produces an individual belonging to one of the above N classes. We will call such a set of classes a "closed biotype." We will suppose only that the probability of obtaining an individual of class j as a result of crossing two individuals of classes i and k has some definite value $A_{ik}^j = A_{ki}^j$, and we will call these probabilities⁴ "heredity coefficients of a given biotype." It follows from the definition of a closed biotype that

$$(1) \quad \sum_{j=1}^N A_{ik}^j = 1.$$

Let α_j be the probability that an individual belongs to class j , then under panmixia⁵ the probability of belonging to class j in the next generation is given by

$$(2) \quad \alpha_j' = \sum_{i,k} A_{ik}^j \alpha_i \alpha_k.$$

¹ The original was published in the *Annales Scientifiques de l'Ukraine*, Vol. 1 (1924), p. 83-114.

² *C. R. Ac. Sc.*, Vol. 177, pp. 528-531, 581-584.

³ Thanks are due to the Brown University Mathematical Library for their loan of this rare periodical.

⁴ A_{ik}^j is, of course, the relative probability that an offspring belong to class j , given that the parents belong to classes i and k .

⁵ That is, complete absence of selection.