From (10) and (14) we conclude that the joint distribution density of the real and imaginary parts of the roots of (9) is given by

$$\left(\frac{1}{\sqrt{2\pi}\,\sigma}\right)^{2n} \exp\left[-\frac{1}{2\sigma^2}\left\{\sum_{j=1}^{n} z_j \sum_{j=1}^{n} \bar{z}_j + \cdots + z_1\bar{z}_1 \cdots z_n\bar{z}_n\right\}\right] \sum_{p=1}^{n} \sum_{q=p+1}^{n} |z_p - z_q|^2. \tag{15}$$

A NOTE ON THE PROBABILITY OF ARBITRARY EVENTS

By Hilda Geiringer[1]

Bryn Mawr College

In a recently published paper [1] on arbitrary events the author studies the probability of the occurrence of at least m among n events. Denoting by $p_m(\gamma_1, \gamma_2, \cdots \gamma_r)$ the probability that at least m among the r events, $E_{\gamma_1}, \cdots E_{\gamma_r}$ occur, and by $p_{[\alpha_1, \alpha_2, \dots, \alpha_r]}$ the probability of the non occurrence of the events numbered $\alpha_1, \alpha_2, \cdots \alpha_r$ and of the occurrence of the $n - r$ others, he proves

$$-p_1(\alpha_{r+1}, \cdots \alpha_n) + \sum_{\gamma_1} p_1(\gamma_1, \alpha_{r+1}, \cdots \alpha_n) - \sum_{\gamma_1}\sum_{\gamma_2} p_1(\gamma_1, \gamma_2, \alpha_{r+1}, \cdots \alpha_n) + \cdots + (-1)^r \sum p_1(1, \cdots n) = p_{[\alpha_1, \cdots \alpha_r]}. \tag{I}$$

(Theorem VI, page 336). From (I) he deduces that *a necessary and sufficient condition* for the existence of a system of events $E_1, \cdots E_n$ associated with given values $t_1(\alpha_1, \cdots \alpha_k)$ is that the expressions on the left side of (I) computed from these t's are $\geqq 0$ for all possible combinations of the α's (Theorem VII). He also points out that it was not possible to find similar (necessary and sufficient) conditions for $m \neq 1$. I wish to show in this note the relation between these theorems and some well known basic facts of the theory of arbitrarily linked events and to add some remarks.

1. Given n chance variables x_i $(i = 1, \cdots n)$ denote by $x_i = 1$ the "occurrence of E_i", by $x_i = 0$ its non occurrence and by $v(x_1, x_2, \cdots x_n)$ the probability of "the result $(x_1, x_2, \cdots x_n)$" i.e., the probability that the first variable equals x_1 the second $x_2, \cdots$ the last x_n; e.g. $v(1, 1, 1, 0, \cdots 0) = v_{[45\dots n]}$ is the probability that only the three first events occur. Hence the v's are 2^n probabilities, arbitrary except for the *condition to have the sum* 1.

Instead of these v's we often introduce another set of $2^n - 1$ probabilities, namely p_i the probability of the occurrence of E_i $(i = 1, \cdots n)$; p_{ij} that of the joint occurrence of E_i and E_j $(i, j = 1, \cdots n)$; $\cdots$ $p_{12\dots n}$ the probability that *all* the events occur.

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