ON THE THEORY OF TESTING COMPOSITE HYPOTHESES WITH ONE CONSTRAINT

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1. Introduction. Our purpose is to extend some of the Neyman-Pearson theory of testing hypotheses to cover certain cases of frequent interest which are complicated by the presence of nuisance parameters. Our results give methods of finding critical regions of types B and B_1. Type B regions were defined by Neyman [1] for the case of one nuisance parameter. Type B_1 regions are the natural generalization of the type A_1 regions of Neyman and Pearson [5] to permit the occurrence of nuisance parameters. The reader familiar with the work of these authors will recognize most of the notation and some of the methods.

We consider a joint distribution of n random variables $x_1, x_2, \cdots, x_n$, depending on l parameters $\theta_1, \theta_2, \cdots, \theta_l$, $l \leqq n$. The functional form of the distribution is given. The random variables may be regarded as the coordinates of a point E in an n-dimensional sample space W, the parameters, as the coordinates of a point Θ in an l-dimensional space Ω of admissible parameter values. Ω, unlike W, in general will not be a complete Euclidian space. Let ω denote the subspace of Ω defined by $\theta_1 = \theta_1^0$. The hypothesis we consider is

$$H_0 : \Theta \epsilon \omega.$$

Neyman and Pearson [4] call H_0 a hypothesis with $l - 1$ degrees of freedom; for our present purpose we shift the emphasis by saying it has one constraint.

It is clear that whenever we test whether a parameter has a given value, and other parameters occur in the distribution, we are testing a hypothesis with one constraint. Hypotheses of the type $\theta_1 = \theta_2$, in which we do not specify the common value of θ_1 and θ_2, nor the values of any other parameters, may always be transformed to H_0 by choosing new parameters. In general, the hypothesis that the parameter point Θ lies on some hypersurface in Ω, $g(\theta_1, \theta_2, \cdots, \theta_l) = g_0$, may be transformed to H_0 if the function g satisfies certain conditions,—say, g is continuous and monotone-increasing in one of the θ's for all Θ in Ω. Another circumstance lending importance to the theory of testing hypotheses with one constraint is its connection with the theory of confidence intervals, which we shall point out below.

The path which led Neyman to critical regions of type B is the following: Every Borel-measurable region w of sample space determines a test of H_0, which consists of rejecting H_0 if and only if E falls in w. In deciding which is a most efficient test, one may limit the competition to similar[1] regions, if such exist. Because of the general non-existence [2, p. 372] of uniformly most

[1] Defined by condition (a) of definition 1.