where
$$u = [\alpha^2(1 - \alpha) - \alpha\sigma_P^2]/\sigma_P^2$$
$$v = [\alpha(1 - \alpha)^2 - (1 - \alpha)\sigma_P^2]/\sigma_P^2.$$

This distribution can now be used to establish tolerance limits. For example, it follows from (1) that for a sample size $n \geq 214$, and a tolerance region given by the ellipse $T^2 = 9.21$, then $E(P) = .99$ and the Prob.$\{.985 \leq P \leq .995\} \geq .992$.

Care must be taken in the use of these and similar results, for if the distribution is not a bivariate normal one, a large error may be introduced which will not be eliminated with increasing n; however the error will probably be small when a tolerance region is found for the means $\bar{x}$, $\bar{y}$ of a future sample of k observations ($k \geq 20$) as contrasted with a tolerance region for a single observation. An exact treatment of the case when the bivariate distribution is unknown has been given by Wald in the present issue of the *Annals of Mathematical Statistics*.

REFERENCES

[1] S. S. Wilks, "Determination of sample sizes for setting tolerance limits," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 91–96.
[2] Harold Hotelling, "A generalization of Student's ratio," *Annals of Math. Stat.*, Vol. 2 (1931), pp. 360–378.

A NEW APPROXIMATION TO THE LEVELS OF SIGNIFICANCE OF THE CHI-SQUARE DISTRIBUTION.

By Leo A. Aroian

Hunter College

Recent articles on the percentage points of the χ^2 distribution [1], [2], have directed my attention to a method proposed in my investigation of Fisher's z distribution [3], a method particularly useful and easily computed for n large. In addition, this method avoids interpolation. If $t = \dfrac{\chi^2 - n}{\sqrt{2n}}$, and $\alpha_3 = \sqrt{\dfrac{8}{n}}$, the measure of skewness for the χ^2 distribution, the following formulas give significance levels of t as quadratic functions of α_3, $t = a + b\alpha_3 + c\alpha_3^2$. The values of a, b, and c were found by the usual method of least squares, fitting each formula to the values of t [4] for $\alpha_3 = 0, \pm 0.1, \pm 0.2, \pm 0.3$, and ± 0.4. Then the value of a in each instance was adjusted to give the proper value of t when $\alpha_3 = 0$: e.g. the constant term by the method of least squares for the 1 per cent point is 2.32633 which we change to 2.32635. The range $|\alpha_3| \leqq .4$ corresponds to $n \geqq 50$, but the formulas are quite satisfactory for $n \geqq 30$. Formulas for t when $|\alpha_3| > .4$ [3] are easily derived, but such results while more accurate in the range