SOME NEW METHODS IN MATRIX CALCULATION[1]

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I. Introduction

1. The increased practical importance of matrix calculation. This paper will be concerned chiefly with methods of finding the inverse of a matrix, and of finding the latent roots and latent vectors, which are also known by a variety of other names associated with particular applications, such as principal axes in geometry and mechanics, and principal components in psychology. These two computational problems are of extremely wide application. The first is closely related to the solution of systems of linear equations, which we shall also consider. In the method of least squares the solution of the normal equations is best carried out with the help of the inverse of the matrix of the coefficients, since at least some of the elements of this inverse matrix are needed in evaluating the results in terms of probability, a vitally necessary step, and since the inverse matrix is useful also in various other ways, such as altering the set of predictors used in a regression equation. Modern statistics also utilizes quadratic and bilinear forms such as the generalized Student ratio [15] for discriminating between samples according to multiple variates instead of one only, the associated discriminant functions [10], the closely related figurative distance of Mahalanobis, Bose and Roy [5] and the critical statistic in an investigation by Wald [28] of the efficient classification of an individual into one of two groups. All these may be calculated very easily from the inverse of a matrix of sums of products, or of covariances or correlations, or from the principal components. Consideration of the relations between two sets of variates [18] may utilize both the inverse of a matrix and a process resembling the calculation of principal components. Similar computational problems arise in applying to sets of numerous variates the contributions to multivariate statistical analysis of R. A. Fisher, S. S. Wilks, W. G. Madow, M. A. Girshick, P. L. Hsu and M. S. Bartlett. Among the non-statistical applications of the inverse matrix and of latent roots and vectors are problems of dynamics, both in astronomy and in airplane design [12], the analysis of stresses and strains in structures [26, 27], and electrical engineering problems [24].

Perhaps no objection to attempts at statistical inference is more common than that the variation of this or that relevant factor has been ignored. For example in dealing with time series the need of allowing for trend and seasonal variation, perhaps by means of a sequence of orthogonal polynomials for trend and of

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