ON FUNDAMENTAL SYSTEMS OF PROBABILITIES OF A FINITE NUMBER OF EVENTS

By Kai Lai Chung

Tsing Hua University, Kunming, China

We consider a probability function $P(E)$ defined over the Borel set of events generated by the n arbitrary events $E_1, \cdots, E_n$, which will be denoted by $\mathfrak{L}(1, \cdots, n)$.

We use the same notations as in the author's former paper[1], with the following abbreviations. We denote a combination $(\alpha_1 \cdots \alpha_a)$ simply by (α), and use the corresponding Latin letter a for its number of members. Similarly we write (β) for $(\beta_1 \cdots \beta_b)$, but (ν) for $(1, \cdots, n)$. We say that (β) belongs to (α) and write $(\beta) \epsilon (\alpha)$ when and only when the set $(\beta_1 \cdots \beta_b)$ is a subset of $(\alpha_1 \cdots \alpha_a)$. Then and then only we write $(\alpha) - (\beta)$ for the subset of elements of (α) that do not belong to (β); thus we may write it as (γ) with $c = a - b$. When and only when (α) and (β) have no common elements, we write $(\alpha) + (\beta)$ for the set of elements that belong either to (α) or to (β); thus we may write it as (γ), with $c = a + b \leqq n$. We note the case for empty sets: $(0) + (0) = (0)$. Now we can write $p_{[(\alpha)]}$ for $p_{[\alpha_1 \cdots \alpha_a]}$, $p_{((\alpha))}$ for $p_{\alpha_1 \cdots \alpha_a}$, $p_b((\alpha))$ for $p_b(\alpha_1 \cdots \alpha_a)$, etc. Further we denote by $p_{[b]}((\alpha))$ $(1 \leqq b \leqq a \leqq n)$ the probability of the occurrence of exactly b events out of $E_{\alpha_1}, \cdots, E_{\alpha_a}$, and write

$$P_a^{(m)}((\nu)) = \sum_{(\alpha) \epsilon (\nu)} p_m((\alpha)), \qquad P_a^{[m]}((\nu)) = \sum_{(\alpha) \epsilon (\nu)} p_{[m]}((\alpha));$$

since a is fixed by the left-hand sides, the summations on the right-hand sides are to be extended to all the $\binom{n}{a}$-combinations of (ν).

A sum written $\sum_{(\beta) \epsilon (\alpha)}$ is to be extended to all combinations (β), $b = 0, 1, \cdots, a$ belonging to (α), when b is not previously fixed; it is to be extended to all the $\binom{a}{b}$-combinations belonging to (α), when b is previously fixed.

Definition 1. *A system of quantities is said to form a fundamental system of probabilities for a set of events if and only if the probability of every event in the set can be expressed in terms of these quantities.*

Definition 2. *An event in $\mathfrak{L}(1, \cdots, n)$ is said to be symmetrical if and only if it is identical with every event obtained by interchanging any pair of suffixes (i, j) $(i, j = 1, \cdots, n)$ in the definition of it.* The subset of symmetrical events in $\mathfrak{L}(1, \cdots, n)$ will be denoted by $\mathfrak{S}(1, \cdots, n)$.

From the normal form[2] of every event in $\mathfrak{L}(1, \cdots, n)$ and the principle of

[1] "On the probability of the occurrence of at least m events among n arbitrary events," *Annals of Math. Stat.*, Vol. 12, 1941.

[2] See Hilbert-Ackermann, *Grundzüge der theoretischen Logik*, Chap. 1.