ON THE PROBLEM OF TESTING HYPOTHESES

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1. Introduction. The following is known as the problem of testing a simple statistical hypothesis. The probability distribution of a variate X depends on a parameter ϑ. In the course of experiments each time a value x of X is observed, one pronounces one of the two assertions: "ϑ equals ϑ_0" or "ϑ is different from ϑ_0." The first assertion is made when the observed value x falls in a "region of acceptance" A, the second, if x falls in the complementary region $\bar{A}$. What is the chance of these assertions being correct and how can A be chosen to make this chance as high as possible?

The distribution for the variate X is considered as given. Let $P(x \mid \vartheta)$ be the probability of the value of X being $\leqq x$. It is obvious that to know $P(x \mid \vartheta)$ is not sufficient for computing the success or error chances of the above assertions. There is another distribution function $P_0(\vartheta)$ involved which we may call the initial or the a priori or the over-all distribution of the parameter ϑ. The meaning of $P_0(\vartheta)$ is as follows. In the infinite sequence of trials there will be among the first N experiences N_1 cases where the assertion that the parameter value is $\leqq \vartheta$ proves correct. Then $P_0(\vartheta)$ is the limit of the ratio N_1/N when N tends to infinity. If N_0 is the number of cases in which the actually pronounced assertions $\vartheta = \vartheta_0$ or $\vartheta \neq \vartheta_0$ respectively, prove correct, the limit of N_0/N is the success chance and of $1 - N_0/N$ the error chance of the test under consideration. It would not make any sense to assume that an error chance exists but the over-all chance $P_0(\vartheta)$ does not.[1]

The success and error chances for the assertions $\vartheta = \vartheta_0$ and $\vartheta \neq \vartheta_0$ depend on both functions $P(x \mid \vartheta)$ and $P_0(\vartheta)$. But in most practical cases nothing or very little is known about the parameter distribution. Usually, only the limits within which ϑ varies are known, or a set of distinct values is given which ϑ can assume. Therefore, the problem of testing a hypothesis must be modified in the following way. We ask: *What can be said about the error and success chances of the two alternative assertions and about the choice of the region of acceptance, if $P_0(\vartheta)$ is entirely or partly unknown?* This form of the question corresponds more or less to the conception generally adopted today.

In section 4 of this paper a complete answer to the question is presented for the case of a parameter distribution that is entirely unknown except for the range of possible ϑ-values. This solution, with the restriction to a parameter assuming distinct values only, was already given by Robert W. B. Jackson in a paper devoted mainly to some genetical problems [1]. The particular circumstances prevailing under the restriction to distinct parameter values will be discussed

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[1] The expression "chance" rather than "probability" is used here since no randomness is required. Cf. the author's paper [2] p. 157.