FURTHER RESULTS ON PROBABILITIES OF A FINITE NUMBER OF EVENTS

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In a recent paper[1] the author has generalized some inequalities of Fréchet to the following:

Let $n \geqq a \geqq m \geqq 1$, and let

$$\binom{n-m}{a-m}^{-1} P_a^{(m)}((\nu)) = A_a^{(m)}.$$

$$\Delta F(a) = F(a) - F(a+1), \qquad \Delta^h F(a) = \Delta(\Delta^{h-1} F(a));$$

then

$$\Delta A_a^{(m)} \geqq 0, \qquad \Delta^2 A_a^{(m)} \geqq 0.$$

Using a generalized Poincaré's formula, P. L. Hsu has improved these inequalities to the recurrence formula stated below.

Hsu's formula is

$$\Delta A_a^{(m)} = \frac{m}{n-m} A_{a+1}^{(m+1)}. \tag{1}$$

Proof: We have

$$p_m((\alpha)) = \sum_{b=m}^{a} (-1)^{b-m} \binom{b-1}{m-1} S_b((\alpha)).$$

For a fixed "a" summing over all $(\alpha) \epsilon (\nu)$,

$$\sum_{(\alpha)\epsilon(\nu)} p_m((\alpha)) = \sum_{b=m}^{a} (-1)^{b-m} \binom{b-1}{m-1}\binom{n-b}{a-b} S_b((\nu))$$

$$A_a^{(m)} = \binom{n-1}{m-1} \sum_{b=m}^{a} (-1)^{b-m} \binom{a-m}{b-m}\binom{n-1}{b-1}^{-1} S_b((\nu))$$

$$\begin{aligned}\Delta A_a^{(m)} &= \binom{n-1}{m-1}\left\{\sum_{b=m}^{a} (-1)^{b-m}\left[\binom{a-m}{b-m} - \binom{a+1-m}{b-m}\right]\binom{n-1}{b-1}^{-1} S_b((\nu)) - (-1)^{a+1-m} \cdot \binom{n-1}{a}^{-1} S_{a+1}((\nu))\right\}\\ &= \binom{n-1}{m-1} \sum_{b=m+1}^{a+1} (-1)^{b-m-1} \binom{a-m}{b-m-1}\binom{n-1}{b-1}^{-1} S_b((\nu))\\ &= \frac{m}{n-m} A_{a+1}^{(m+1)}, \qquad \text{Q.E.D.}\end{aligned}$$

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[1] "On the probability of the occurrence of at least m events among n arbitrary events," *Annals of Math. Stat.*, Vol. 12 (1941), pp. 328–338. We use throughout the same notation used in this paper, and that referred to in footnote 3.