ON THE CONSTRUCTION OF SETS OF ORTHOGONAL LATIN SQUARES[1]

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1. Introduction.

An m-sided Latin square is an arrangement of m symbols into a square in such a way that no row and no column contains any symbol twice. Two Latin squares are called orthogonal if, when one is superimposed upon the other, every pair of symbols occurs only once. For instance the squares

$$\begin{array}{ccc} A & B & C \\ B & C & A \\ C & A & B \end{array} \qquad \begin{array}{ccc} \alpha & \beta & \gamma \\ \gamma & \alpha & \beta \\ \beta & \gamma & \alpha \end{array}$$

are orthogonal. The resulting square is

$$\begin{array}{ccc} A\alpha & B\beta & C\gamma \\ B\gamma & C\alpha & A\beta \\ C\beta & A\gamma & B\alpha. \end{array}$$

A pair of orthogonal Latin squares is called a Graeco-Latin square. A method has not yet been found by which all possible sets of mutually orthogonal squares can be constructed. However, methods are available for constructing certain special sets, and although we cannot obtain all possible sets with these methods they yield a great variety of designs.

To understand these methods we shall have to use certain fundamental concepts of the theory of numbers. In the following we shall deal therefore only with integers and all symbols used will denote only integers.

Let a, b, m denote certain integers. We say

$$a \equiv b \ (m),$$

(in words a is congruent to b modulo m) if $a - b$ is divisible by m.

Such congruences can be treated like equations. For instance: If $a \equiv b \ (m)$, then $a \pm c \equiv b \pm c \ (m)$, $ac \equiv bc \ (m)$. The proofs of these statements are obvious from the definition of $a \equiv b \ (m)$.

If $a \equiv b \ (m)$, and $c \equiv d \ (m)$, then $ac \equiv bd \ (m)$, and $a \pm c \equiv b \pm d \ (m)$.

Proof: According to our definition we have

$$a - b = \lambda_1 m \qquad a = b + \lambda_1 m$$

$$c - d = \lambda_2 m \qquad c = d + \lambda_2 m$$

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