# MULTIPLE SAMPLING WITH CONSTANT PROBABILITY

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**1. Introduction.** In an attempt to reduce inspection costs, manufacturers have frequently resorted to sampling procedure in which the disposition of an aggregate or lot of similar units does not necessarily depend upon the results of a single sample. In practice, however, the number of permissible additional samples is limited to one or two; nevertheless, if the lot is very large, an appreciable reduction in the expected sample may be accomplished by allowing a greater number of additional samples. In this article probability formulae will be derived for an inspection procedure for infinite lots in which the number of additional samples is not limited and may be any number depending upon the results of the sampling. This development will be limited to the simple case of attribute inspection in which the units fall into two categories—satisfactory units or defective units. If $p$ denotes the fraction defective in an infinite lot, then the probability of finding exactly $m$ defective units or defects in a sample of $n$ is

$$P(m, n) = \binom{n}{m} p^m q^{n-m}, \qquad q = 1 - p. \tag{1}$$

Since $P(m, n)$ is the probability of $m$ successes in $n$ trials with constant probability of success $p$, though the terminology of commercial inspection will be used in this article, the results are applicable to other situations involving repeated trials with constant probability of success.

In contrast with multiple sampling, a single sample inspection procedure for lots of the type here considered is one in which a lot of units is accepted or rejected on the basis of the number of defective units found in the sample. Thus a lot is accepted if the number of defects is at most an integer $c$ the "acceptance number," and rejected if the number exceeds $c$. For an infinite lot containing a fraction $p$ of defects and a sample of $n$ units, the probability of accepting is by (1)

$$\Pi_s (c, n) = \sum_{m \leqq c} P(m, n), \tag{2}$$

and the probability for rejection is the difference between this sum and unity.

**2. Multiple sampling.** The procedure in multiple sampling is to examine first an initial sample of $n_0$ units. If the number of defects in this initial sample is at most $c$ the lot is accepted and if the number of defects exceeds $c + k$ ($k$ an integer) the lot is rejected. But if the number of defects is greater than $c$ and less than $c + k + 1$ an additional sample is removed and examined. In the latter case similar criteria determine whether the lot is to be accepted or rejected or this method of sampling continued. With an infinite lot this scheme of samp-