ON THE STATISTICS OF SENSITIVITY DATA

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1. Introduction. "Sensitivity data" is a general term for that type of experimental data for which the measurement at any point in the scale destroys the sample; as a consequence, new samples are required for each determination. Examples of such data occur in biology in dosage-mortality determinations, in psychophysics in questions concerning sensitivity responses, and, more recently, in the theory of solid explosives, in questions concerning the sensitivity of explosive or detonative mixtures.

Methods of analyzing such data have been discussed by Bliss[1] and Spearman[2], and others. The present paper is a generalization of Spearman's result; it is the feeling of the authors that Spearman's method, if properly founded in mathematical theory, is preferable to Bliss', for it does not necessitate the assumption of some type of distribution prior to analysis, and hence resembles the standard treatment of independent observations made on the same object.

Throughout the following discussion, we let x_i be the magnitude of a certain "stimulus" (be it dosage, physical stimulus, or strength of blow) and p_i the corresponding fraction of objects unaffected by the stimulus. Bliss' method consisted in assuming that the p_i represented the cumulative distribution of some known function (in his case, the normal function), and hence the p_i could be transformed into a variable t_i linearly dependent on the x_i. The difficulty of this treatment, in addition to the distribution assumption, lies in the fact that the t_i do not have equal standard errors, and the straight line fit is very cumbersome.

Instead, Spearman makes the much simpler assumption that if p_i is unaffected at x_i, and p_{i+1} at x_{i+1}, then $p_i - p_{i+1}$ is an estimate of the fraction that is just affected (i.e., the fraction of those that have "critical" responses) at about $\frac{1}{2}(x_i + x_{i+1})$. If the x_i are evenly spaced, as we shall assume them to be throughout, and $p_1 = 1.0$ and $p_n = 0$, then any set of sensitivity data may be transformed into a set of data on critical responses classified into classes whose midpoints are evenly spaced. Without loss of generality, we shall assume the x_i's to be integers and the intervals to be unity. The data on critical responses can then be treated in the normal way, and $\bar{X}$ and all the measures of dispersion calculated in the usual fashion. In order to justify such procedures, however, it is necessary to show how the sampling errors of $\bar{X}$ and the higher moments can be estimated.

[1] C. I. Bliss, "The calculation of the dosage mortality curve," *Annals of Applied Biology*, Vol. 22, pp. 134–167.

[2] C. Spearman, "The method of 'right and wrong cases' (constant stimuli) without Gauss' formulae," *British Jour. of Psych.*, Vol. 2, 1908, pp. 227–242.