FURTHER CONTRIBUTIONS TO THE PROBLEM OF SERIAL CORRELATION

By Wilfrid J. Dixon

Princeton University

1. Introduction. Recently, there has been an increasing interest in the study of the serial correlation of observations. The development of the distribution theory and significance criteria was retarded by the fact that the successive differences or successive products of statistical variates are not independent. However, these difficulties have been overcome to a considerable extent by recent work of several authors. In order to indicate the nature of the contributions embodied in the present paper, it will be necessary to describe rather precisely the contributions of these authors.

Suppose $x_1, x_2, \cdots, x_n$ are n independent observations of a random variable x which is normally distributed with mean a and variance σ^2. Let us define

$$\begin{aligned} \delta^2_{n-1} &= \sum_{i=1}^{n-1} (x_{i+1} - x_i)^2 & \delta^2_n &= \sum_{i=1}^{n} (x_{i+1} - x_i)^2 \\ C_{n-1} &= \sum_{i=1}^{n-1} (x_i - \bar{x})(x_{i+1} - \bar{x}) & C_n &= \sum_{i=1}^{n} (x_i - \bar{x})(x_{i+1} - \bar{x}) \qquad (1.1) \\ {}_LC_n &= \sum_{i=1}^{n} (x_i - \bar{x})(x_{i+L} - \bar{x}) & V_n &= \sum_{i=1}^{n} (x_i - \bar{x})^2 \end{aligned}$$

in which $x_{n+i} = x_i$. The ratio of any of the first five values to V_n will be a measure of the relation between the successive observations x_i.

Von Neumann [2] has studied the ratio $\eta = \delta^2_{n-1}/V_n$. He obtains an expression for the sampling distribution of the ratio η. He solves the equivalent problem of determining the distribution of $\sum_{i=1}^{n-1} A_i y_i^2$ where the point $(y_1, y_2, \cdots, y_{n-1})$ is uniformly distributed over the spherical surface $\sum_{i=1}^{n-1} y_i^2 = 1$ and the A_i are the characteristic values of δ^2_{n-1}. He obtains the distribution $\omega(\gamma)$ of $\gamma = \sum_{i=1}^{m} B_i x_i^2$ (m even) where the point $(x_1, x_2, \cdots, x_n)$ is uniformly distributed over the spherical surface $\sum_{i=1}^{m} x_i^2 = 1$ and $B_1 \geq B_2 \geq \cdots \geq B_m$. $\omega(\gamma)$ is found by solving the equation

$$\int_{B_m}^{B_1} (\gamma - z)^{-\frac{1}{2}m} \omega(\gamma)\, d\gamma = \prod_{i=1}^{m} (B_i - z)^{-\frac{1}{2}}. \qquad (1.2)$$

The distribution of η is then a special case of this distribution. The first four moments were obtained by Williams [5] by the use of a generating function. In