NOTES

This section is devoted to brief research and expository articles on methodology and other short items.

ON THE EXPECTED VALUES OF TWO STATISTICS

By H. E. Robbins

Post Graduate School; Annapolis, Md.

In a previous paper[1], the following theorem was proved. Let X be a random, Lebesgue measurable subset of Euclidean m dimensional space E_m, and let $\mu(X)$ be the measure of X. For every point x of E_m let $p(x)$ be the probability that X contains x. Then

$$E(\mu(X)) = \int_{E_m} p(x)\, d\mu(x). \tag{1}$$

In the present note we shall show how this theorem may be used to find the expected values of two statistics which arise in sampling theory. Applications to similar problems may suggest themselves to the reader.

Let Y be a real random variable with c. d. f. (cumulative distribution function) $\sigma(y)$, so that for every y,

$$\Pr\ (Y < y) = \sigma(y). \tag{2}$$

Let $Y_1, \cdots, Y_n$ be n independent random variables, each with the distribution of Y. Finally, let

$$\begin{aligned} A &= \min\,(Y_1, \cdots, Y_n), \\ B &= \max\,(Y_1, \cdots, Y_n), \\ R &= B - A, \\ F &= \sigma(B) - \sigma(A). \end{aligned} \tag{3}$$

Although the values of $E(F)$ and $E(R)$ can be found from the sampling distributions of F and R, and, in fact, are well known, we shall show how to apply (1) to find $E(F)$ and $E(R)$ directly.

To find the first of these, let X denote the set of points in the interval $0 \leq x \leq 1$ such that

$$\sigma(A) < x < \sigma(B). \tag{4}$$

Then X is a random set with measure

$$\mu(X) = F. \tag{5}$$

Moreover, for any point x the probability that X shall contain x is clearly

$$p(x) = 1 - x^n - (1 - x)^n. \tag{6}$$