SOME COMBINATORIAL FORMULAS WITH APPLICATIONS TO PROBABLE VALUES OF A POLYNOMIAL-PRODUCT AND TO DIFFERENCES OF ZERO

By L. C. Hsu

National Southwest Associated University, Kunming, China

1. The main purpose of the paper is to establish some combinatorial formulas concerning the mathematical expectations or probable values of a product of n given polynomials. The problem may be stated more definitely as follows:

Let $x_1, \cdots, x_n$ be n non-negative discontinuous variables for which we have assumed that the probability that each x takes a possible value is equally likely, and let $f_1(x), \cdots, f_n(x)$ be n given polynomials. Then we shall ask: What is the probable value of the product $f_1(x_1) \cdots f_n(x_n)$, provided the sum of the variables $x_1, \cdots, x_n$ is known? More generally, we may consider the problem with certain restrictions to x such as $a \leq x_i \leq b$, $(i = 1 \cdots n)$.

By a limiting process[1] it will be found that all the formulas established for the preceding problem can be extended to the case of continuous variables. On this account, it is important to find explicit formulas for the problem merely involving discontinuous variables.

By the definition[2] of MacMahon, we say that a set of numbers $(x_1 \cdots x_n)$ is over all different compositions of m into n parts with each $x \geq k$, if $(x_1 \cdots x_n)$ runs over all different integer solutions of the linear equation $x_1 + \cdots x_n = m$ with each $x \geq k$. We shall use the notation $(m; k; x_1 \cdots x_n)$, or simply $(m; k; x)$, to denote that a set of numbers $(x_1 \cdots x_n)$ is over all different compositions of m into n parts with each $x \geq k$.

The notation $E(m; \delta; [f_1(x)] \cdots [f_n(x)])$ will be used to denote the mathematical expectation of the product $f_1(x_1) \cdots f_n(x_n)$ in which the sum of n variable quantities $x_1, \cdots, x_n$ is known, namely $x_1 + \cdots + x_n = m$, and each quantity is a multiple of δ and m is of course a multiple of δ. Thus by the definition[3] of mathematical expectations we have

$$E(m; \delta; [f_1] \cdots [f_n]) = \Big(\sum_{(m/\delta;\, 1;\, x)} 1 \Big)^{-1} \sum_{(m/\delta;\, 1;\, x)} f_1(x_1 \delta) \cdots f_n(x_n \delta), \tag{1}$$

where the summation on the right-hand side runs over all different compositions of m/δ into n parts with each $x \geq 1$, and the given constant δ is called a "varying unit", that is the least possible difference between two unequal quantities in $(x_1 \cdots x_n)$. If the varying unit approaches zero, $(x_1 \cdots x_n)$ will become a set of continuous variables.

---

[1] The limiting process will be illustrated by the proof of corollary 2 of theorem 1 in this paper.

[2] MacMahon, *Combinatory Analysis*, Vol. 1, p. 150.

[3] See for example W. Burnside, *Theory of Probability*, Chap. 4, 13.