ERROR CONTROL IN MATRIX CALCULATION

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1. Introduction. The solutions of large sets of simultaneous equations and the inversion of matrices are often complicated by the fact that errors, such as those introduced by rounding, become magnified in the course of the calculations to such an extent that the results are useless. In this paper we shall show that if the norm of the matrix $A - I$ is less than 0.35, operations involving the inversion A or the multiplication by A^{-1} will be in a state of error control for "Doolittle" methods of calculation. Thus such calculations may be carried through with assurance that the errors in the results will be limited to two or three significant figures. We also point out that as soon as an approximation to A^{-1} is available, most problems may be restated to bring them within the requirements for error control. Therefore the solution can be immediately completed to the desired degree of accuracy in one step instead of requiring multiple steps as do the iterative methods.

2. The inversion of special matrices. Consider the problem of inverting the matrix $(I + F)$ where I is the identity matrix and $(I + F)$ is a non-singular square matrix. Let

(2.1) $$G = (I + F)^{-1}.$$

Then

(2.2) $$(I + F)G = I$$

or

(2.3) $$G = I - FG.$$

In ordinary algebra this would not be a practical formula for the calculation of G. However in matrix algebra the situation may be different. Examine the expanded form of G:

(2.4) $$g_{ij} = \delta_{ij} - \Sigma f_{ik} g_{kj}.$$

The summation is over all values of k from 1 to n. Next examine the affect of imposing certain restrictions on F. For example, let $f_{ij} = 0$ if $j \geq i$. This is equivalent to making the summation in (2.4) over the range 1 to $i - 1$. The first row of (2.4) then becomes

$$g_{ij} = \delta_{ij}$$

and no g's appear on the right. For the second row

$$g_{2j} = \delta_{2j} - f_{21} g_{1j}.$$