ON A PROBLEM OF ESTIMATION OCCURRING IN PUBLIC OPINION POLLS

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To arrive at an estimate of the number of electoral votes that will be cast for a presidential candidate a poll is taken of $\lambda_i N$ interviews in the ith state ($i = 1, \cdots, 48$) where the λ_i are fixed constants > 0 such that $\Sigma\lambda_i = 1$ and the respondent is asked for which candidate he intends to cast his vote. To estimate the number of electoral votes which candidate A will receive, the electoral votes of all the states in which the poll shows a majority for candidate A are added and their sum is used as an estimate for the number of electoral votes which candidate A will receive. In this paper certain properties of this estimate will be discussed. It will be shown that it is a biased but consistent estimate and an upper bound for the bias will be derived. Finally we shall derive that distribution of interviews which minimizes the variance of our estimate.

In all that follows we shall consider the poll as a random or stratified random sample and shall disregard the bias introduced by inaccurate answers. Our results however remain valid as long as the sampling variance is proportional to $\frac{1}{\sqrt{N}}$.

We shall use the following notation:

π_i = proportion of voters in the ith state who intend to vote for candidate A.

$$\epsilon_i = 1 \quad \text{if } \pi_i > \tfrac{1}{2}$$
$$\qquad 0 \quad \text{if } \pi_i < \tfrac{1}{2}$$

w_i = number of electoral votes of the ith state.

p_i, e_i = sample values of π_i and ϵ_i resp.

We shall further exclude the case $\pi_i = \frac{1}{2}$.

The number of electoral votes for candidate A is then given by

$$\sum_{i=1}^{i=48} \epsilon_i w_i = \Gamma.$$

As an estimate of Γ we use the quantity

$$\sum_{i=1}^{i=48} e_i w_i = G. \tag{1}$$

Let ρ_i be the probability that $p_i > \frac{1}{2}$ and hence $e_i = 1$. Let $\lambda_i N = N_i$ be the number of interviews in the ith state. If N_i is not too small then ρ_i is given by

$$\rho_i = \int_{\frac{1}{2}}^{\infty} \frac{1}{\sqrt{2\pi}\sigma_i} e^{-(x-\pi_i)^2/2\sigma_i^2}\, dx. \tag{2}$$