

ON AN EXTENSION OF THE CONCEPT OF MOMENT WITH APPLICATIONS TO MEASURES OF VARIABILITY, GENERAL SIMILARITY, AND OVERLAPPING¹

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1. Introduction. Given a frequency distribution $D: [X_i, F_i]$ ($i = 1, 2, 3, \dots, n$), we shall call the expression

$$M_r(D, X_j) = \sum_{i=1}^n (X_i - X_j)^r F_i$$

the r th total moment of D about the origin X_j . We shall consider the weighted sum

$$\mathfrak{M}_r = \sum_j W_j M_r(D, X_j)$$

where W_j denotes the weight corresponding to the particular origin X_j , and the summation is over a field ϕ . In particular, if ϕ is the set of all values assumed in D by the variate X_i , and if $W_j = F_j$, we shall call the quantity the r th *complete total moment* of D . If, on the contrary, W_j is the frequency F'_j of the value X'_j in a second frequency distribution $D': [X'_j, F'_j]$ and ϕ' is the set of all values assumed by the variate X'_j in D' , \mathfrak{M}_r will be called the r th *aggregate moment* of D and D' . A modification of this procedure leads to what we shall call the *moment of transvariation* of D and D' .

The consideration of complete moments draws attention to certain previously known measures of variability which are independent of the origin selected, and also provides simple methods of computation which are useful for data given in the form of a frequency distribution. The investigation of aggregate moments and moments of transvariation gives rise to certain measures of general similarity between two distributions, as well as measures of the amount of overlapping.

2. Sliding and complete moments of a frequency distribution.

2.1. We shall give the name *sliding total moments* of order r to the successive values, for particular values of j , of the expression

$$(2.11) \quad M_r(X_j) = F_j \sum_{i=1}^n [(X_i - X_j)^r F_i].$$

¹The Portuguese original of this paper was written in Brazil, in August 1943. Its translation into English was entirely revised by Dr. T. Greville, Bureau of the Census, who proposed also many simplifications in the derivation of formulae. For his painstaking labor and interest I wish to express my very sincere appreciation. I also wish to thank Dr. W. Edwards Deming for reading the manuscript and making several valuable suggestions.