ON A TEST FOR RANDOMNESS BASED ON SIGNS OF DIFFERENCES[1]

By Henry B. Mann

Ohio State University

1. Introduction. It has been pointed out by J. Wolfowitz [1] that we cannot expect a test for randomness to be most powerful with respect to every possible alternative. It is therefore necessary to find tests designed to distinguish a random sample of observations from the same population from a sample coming from some particular class Ω of distributions. Such a test need be consistent in the sense of Wald and Wolfowitz [2] only with respect to alternatives in the class Ω.

Let $x_1, \cdots, x_n$ be the measurable quality characteristics of n units of a manufactured article. We shall assume that the distribution of x_i is continuous. According to Shewhart the production process is termed "under statistical control" if $x_1, \cdots, x_n$ can be regarded as a random sample of n independent items each coming from the same population with known or unknown distribution function.

In a random sample $p_i = p(x_i > x_{i+1}) = \frac{1}{2}$, where $P(E)$ denotes the probability that E will hold. The class Ω of alternatives which we shall consider is described as follows. The cumulative distribution of x_i is f_i and the f_i, $i = 1, 2, \cdots$, are such that

$$p_i = \tfrac{1}{2} + \epsilon_i, \qquad \sum_{i=1}^{i=n} \epsilon_i = \lambda_n(n-1), \qquad \liminf_{n\to\infty} \lambda_n = \lambda > 0.$$

Such a situation may, for instance, obtain of the production process is under statistical control except for occasionally but not too infrequently occurring periods during which the quality of the product decreases, after which decrease statistical control is immediately restored. If the decreases in quality are sharp enough or the periods of decrease long enough, then the alternative will belong to the class Ω described before.

To give a practical example; consider a drill, which after some period of use will wear off so that the quality of the manufactured article will decrease until the drill is exchanged. After replacement of the drill by a new one, statistical control is immediately restored. Now, if the drill is not replaced in time, the periods of decrease in quality will be long and the rate of decrease will become rapid so that the sequence of distribution functions will satisfy the conditions of the class Ω. A similar situation occurs also in time studies. For instance, in the foregoing example, the time necessary for drilling one hole will tend to increase when the drill is too long in use.

The following test first proposed by Moore and Wallis [3] for the study of

[1] Research under a grant of the Research Foundation of the Ohio State University.