Next, let for $n > e$

$$a_n = n \log^{-\eta} n. \tag{12}$$

Then (8) holds and from (9) and (10) we obtain easily for large n

$$b_n = \sum_{k=1}^{n} \{1 - \log^{-\eta} a_k\} < n - (1 - \epsilon)a_n . \tag{13}$$

Substituting into (7) one sees that, again for sufficiently large n,

$$\Pr \{S_n - n + (1 - \epsilon)a_n < \epsilon a_n\} \to 1, \tag{14}$$

or, since $M = 1$,

$$\Pr \{S_n - nM < -(1 - 2\epsilon)a_n\} \to 1. \tag{15}$$

This proves (I).

A NOTE ON RANK, MULTICOLLINEARITY AND MULTIPLE REGRESSION[1]

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Let $X_{it}(i = 1, 2 \cdots M)$ be set of M random variables, each being observed at $t = 1, 2 \cdots N$. $X_{it} = M_{it} + y_{it}$. (This is essentially the situation envisaged by Frisch [1]). The systematic part of our variables $M_{it} = EX_{it}$. The y_{it} are normally distributed with means zero. Their variances and covariances are independent of t. The M_{it} and y_{it} are independent of each other. Define $\bar{X}_i = \Sigma_t X_{it}/N$ the arithmetic mean of X_{it} and $x_{it} = X_{it} - \bar{X}_i$ the deviation from the mean. Then $a_{ij} = \Sigma_t x_{it}x_{jt}/(N - 1)$ gives the variances and covariances of the observations. We want to determine the rank of the matrix of the variances and covariances of M_{it}.

Now assume that $||V_{ij}||$ is an estimate of the variance-covariance matrix of the error terms or "disturbances" y_{it}. The elements of this matrix are distributed according to the Wishart distribution and are independent of the M_{it}. They can be estimated as deviations from polynomial trends, as deviations from Fourier series, by the Variate Difference Method, etc. The estimates could also be based upon a priori knowledge if for instance the y_{it} are interpreted as errors of measurement. Assume that the estimate is based upon N' observations.

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