ON THE POWER FUNCTIONS OF THE E^2-TEST AND THE T^2-TEST

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1. The general linear hypothesis. Every linear hypothesis about a p-variate normal population or several such populations having common variances and covariances is reducible to the following canonical form [4]: The sample distribution, when nothing whatever has been discarded from the whole sample, being

$$(2\pi)^{-\frac{1}{2}p(m+n)} \mid \alpha_{ij} \mid^{\frac{1}{2}(m+n)} \exp\left\{-\tfrac{1}{2} \sum_{i,j=1}^{p} \alpha_{ij} \sum_{r=1}^{m} (y_{ir} - \eta_{ir})(y_{jr} - \eta_{jr}) - \tfrac{1}{2} \sum_{i,j=1}^{p} \alpha_{ij} \sum_{s=1}^{n} z_{is} z_{js}\right\} \Pi\, dy\, dz \tag{1}$$

$$(n \geq p),$$

where the η_{ir} and the α_{ij} are unknown, the hypothesis to be tested is

$$H: \quad \eta_{ir} = 0 \qquad (i = 1, \cdots, p; r = 1, \cdots, n_1, n_1 \leq m).$$

It is clear that the y_{ir} $(i = 1, \cdots, p;\ r = n_1+1, \cdots, m)$ can have no use. Also, the only useful quantities supplied by the set z_{is} are the statistics

$$b_{ij} = \sum_{s=1}^{n} z_{is} z_{js},$$

because the remaining quantities may be regarded as a set of angles which are independent of y_{ir} and the b_{ij} and which has a known distribution free from any unknown parameter in (1), [2]. After discarding the irrelevant y's and the angles there results the reduced sample distribution

$$K \mid \alpha_{ij} \mid^{\frac{1}{2}(n_1+n)} \mid b_{ij} \mid^{\frac{1}{2}(n-p-1)} \exp\left\{-\tfrac{1}{2} \sum_{i,j=1}^{p} \alpha_{ij} \cdot \sum_{r=1}^{n_1} (y_{ir} - \eta_{ir})(y_{jr} - \eta_{jr}) - \tfrac{1}{2} \sum_{i,j=1}^{p} \alpha_{ij} b_{ij}\right\} \Pi\, dy\, db.$$

Hereafter the indices i, j and r shall have the following ranges:

$$i, j = 1, \cdots, p, \qquad r = 1, \cdots, n_1,$$

and the convention that repetition of an index indicates summation will be adopted. Writing

$$a_{ij} = y_{ir} y_{jr}, \qquad c_{ij} = a_{ij} + b_{ij},$$

we obtain the distribution of the y_{ir} and the c_{ij}:

$$K \mid \alpha_{ij} \mid^{\frac{1}{2}(n_1+n)} \mid c_{ij} - a_{ij} \mid^{\frac{1}{2}(n-p-1)} \exp\left(-\tfrac{1}{2}\alpha_{ij} c_{ij} + \alpha_{ij} y_{ir} \eta_{jr} - \tfrac{1}{2}\alpha_{ij} \eta_{ir} \eta_{jr}\right) \Pi\, dy\, dc. \tag{2}$$