A TWO-SAMPLE TEST FOR A LINEAR HYPOTHESIS WHOSE POWER IS INDEPENDENT OF THE VARIANCE

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1. Introduction. In a paper in the *Annals of Mathematical Statistics*, Dantzig [1] proves that, for a sample of fixed size, there does not exist a test for Student's hypothesis whose power is independent of the variance. Here, a two-sample test with this property will be presented, the size of the second sample depending upon the result of the first. The problem of determining confidence intervals, of preassigned length and confidence coefficient, for the mean of a normal distribution with unknown variance is solved by the same procedure. These considerations including the non-existence of a single-sample test whose power is independent of the variance, are extended to the case of a linear hypothesis. In order to make the power of a test or the length of a confidence interval exactly independent of the variance, it appears necessary to waste a small part of the information. Thus, in practical applications, one will not use a test with this property, but rather a test which is uniformly more powerful, or an interval of the same length, whose confidence coefficient is a function of σ, but always greater than the desired value, the difference usually being slight, at the same time reducing the expected number of observations by a small amount.

Any two sample procedure, such as that discussed in this paper, can be considered a special case of sequential analysis developed by Wald [5].

The problem of whether these tests and confidence intervals are in any sense optimum is unsolved. It is difficult even to formulate a definition of an optimum among sequential tests of a hypothesis against multiple alternatives. However it is shown that, if the variance and initial sample size are sufficiently large, the expected number of observations differs only slightly from the number of observations required for a single-sample test when the variance is known. It also seems likely that the confidence intervals do possess some optimum property among the class of all two-sample procedures.

Although Student's hypothesis is a special case of a linear hypothesis, it is treated separately, because it illustrates the basic idea without any complicated notation or new distributions. The test for Student's hypothesis involves the use only of Student's distribution, even for the power of the test, while the power function of the test proposed here for a linear hypothesis involves a new type of non-central F-distribution.

The notation χ_n^2 is used as a generic symbol for a random variable equal to the sum of squares of n independently normally distributed random variables with mean 0 and variance 1, i.e., χ_n^2 has the χ^2 distribution with n degrees of freedom,

243