and take the values 1 (with probability 2/3) and -2 (with probability 1/3). $X_1 + X_2 + X_3$ takes the values 3 (with probability 8/27), 0 (with probability 12/27), -3 (with probability 6/27) and -6 (with probability 1/27). Hence $E(|X_i|) = 4/5$, and $E(|X_1 + X_2 + X_3|) = 48/27 = 16/9 = 4/3E(|X_i|)$, which is not $\geq 3/2E(|X_i|)$.

REFERENCE

[1] Z. W. Birnbaum and Herbert S. Zuckerman, "An inequality due to H. Hornich," *Annals of Math. Stat.*, Vol. 15 (1944), pp. 328–329.

ON THE INDEPENDENCE OF THE EXTREMES IN A SAMPLE[1]

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In a previous article [1] the assumption was used that the mth observation in ascending order (from the bottom) and the mth observation in descending order (from the top) are independent variates, provided that the rank m is small compared to the sample size n. In the following it will be shown that this assumption holds for the usual distributions.

Let x be a continuous, unlimited variate, let $\Phi(x)$ be the probability of a value equal to, or less than, x; let $\varphi(x)$ be the density of probability, henceforth called the initial distribution. The mth observation from the bottom is written ${}_mx$ and the kth observation from the top is written x_k. Thus, the bivariate distribution $\mathfrak{w}_n({}_mx, x_k)$ of ${}_mx$ and x_k, is such that there are $m - 1$ observations less than ${}_mx$; $k - 1$ observations greater than x_k and $n - m - k$ observations between ${}_mx$ and x_k.

For simplicity's sake write

$$\Phi({}_mx) = {}_m\Phi; \qquad \Phi(x_k) = \Phi_k.$$

$$\varphi({}_mx) = {}_m\varphi; \qquad \varphi(x_k) = \varphi_k.$$

Then

$$\mathfrak{w}_n({}_mx, x_k) = C\,{}_m\Phi^{m-1}\,{}_m\varphi(\Phi_k - {}_m\Phi)^{n-m-k}\varphi_k(1 - \Phi_k)^{k-1}, \tag{1}$$

where

$$C = \frac{n!}{(m-1)!(k-1)!(n-m-k)!}. \tag{1'}$$

In the expression (1) no assumption about dependence or independence of ${}_mx$ and x_k is implied except that these values are taken from the same population.

The distribution (1) is now modified by introducing three conditions. First,

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