NOTES

This section is devoted to brief research and expository articles, notes on methodology and other short items.

COMPUTATION OF FACTORS FOR TOLERANCE LIMITS ON A NORMAL DISTRIBUTION WHEN THE SAMPLE IS LARGE[1]

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In their paper [1], Wald and Wolfowitz discuss the problem of finding tolerance limits of the form $\bar{x} \pm \lambda s$ for a normal distribution. They propose the following large sample formula for λ which appears to be satisfactory for all practical purposes for $N \geqq 2$!

$$\lambda = \sqrt{\frac{n}{\chi_\beta^2}}\, r\left(\frac{1}{\sqrt{N}}, \gamma\right) \tag{1}$$

where N is the number of observations $(n = N - 1)$, γ is the tolerance coefficient, β is the confidence coefficient, r is defined by

$$\frac{1}{\sqrt{2\pi}} \int_{(1/\sqrt{N})-r}^{(1/\sqrt{N})+r} e^{-t^2/2}\, dt = \gamma$$

and χ_β^2 has the property that $P(\chi^2 > \chi_\beta^2) = \beta$ for n degrees of freedom. To compute λ, tables [2] or known approximations [3] for χ_β^2 are customarily used, but the computation of r, even for large N, is tedious, involving an iterative procedure. The purpose of this note is to obtain an expansion of r in terms of $1/\sqrt{N}$ and to combine this expansion with a known one for χ_β^2 to obtain an asymptotic formula for λ.

To derive a large sample formula for r, consider the function

$$f(x, y) = \frac{1}{\sqrt{2\pi}} \int_{x-y}^{x+y} e^{-t^2/2}\, dt - \gamma = 0 \tag{2}$$

where for convenience $\dfrac{1}{\sqrt{N}}$ and r are replaced by x and y. It is desired to express y as a power series in x. Let y_0 be defined by $f(0,y_0) = 0$. Since $f(x,y)$ is a con-

[1] This paper reports work done in the Statistical Research Group, Division of War Research, Columbia University, under Contract OEMsr-618 with the Applied Mathematics Panel, National Defense Research Committee, Office of Scientific Research and Development. The work was first reported in an unpublished memorandum, "Computation of Factors for Tolerance Limits when the Sample is Large" (SRG No. 559, September 24, 1945). A brief account of the application of tolerance limits, including tables, will be published in *Techniques of Statistical Analysis* described in the footnote on page 217.