It can then be shown[4] that

$$\lim_{T\to\infty} \frac{1}{T} \sum_j \lambda_j^2 = 2\pi \int_{-\infty}^{\infty} B^2(u)\, du = \int_{-\infty}^{\infty} \rho^2(\tau)\, d\tau$$

and

$$\lim_{T\to\infty} \frac{1}{T} \sum_j \lambda_j^3 = (2\pi)^2 \int_{-\infty}^{\infty} B^3(u)\, du.$$

It follows now by standard methods that the characteristic function of

$$\frac{1}{\sqrt{T}} \left\{ \int_0^T x^2(t)\, dt - T \right\} \tag{11}$$

approaches, as $T \to \infty$,

$$\exp\left(-\frac{\sigma^2}{2}\xi^2\right),$$

where

$$\sigma^2 = \int_{-\infty}^{\infty} \rho^2(\tau)\, d\tau.$$

Thus, as $T \to \infty$, the distribution of (11) becomes normal with mean 0 and variance σ^2.

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APPROXIMATE FORMULAS FOR THE RADII OF CIRCLES WHICH INCLUDE A SPECIFIED FRACTION OF A NORMAL BIVARIATE DISTRIBUTION

By E. N. Oberg

University of Iowa

1. Introduction. Given the normal bivariate error distribution

$$\phi(x, y) = (1/2\pi\sigma_x\sigma_y)e^{-(x^2/2\sigma_x^2+y^2/2\sigma_y^2)}. \tag{1}$$

The purpose of this paper is to present certain approximate formulas for the radii of circles whose centers are at the origin, which include a prescribed proportion, p, of errors. The formulas are, for given σ_x, σ_y, and p,