with

$$W_i = \tfrac{1}{2}[(x_i - x_0)^2(\xi_i - x_0)f_{xxx}(\xi_i', \eta_i') + (x_i - x_0)^2(\eta_i - y_0)f_{xxy}(\xi_i', \eta_i')$$
$$+ 2(x_i - x_0)(y_i - y_0)f_{xy}(\xi_i, \eta_i) + (y_i - y_0)^2 f_{yy}(\xi_i, \eta_i)].$$

Corresponding formulas can be derived in this way for any value of n; in fact, several alternatives may be obtained in each case. In all cases the error $f(x_0, y_0)$ is given in terms of the derivatives of g alone if a polynomial of a certain type is used for the interpolating function. For equation (4), the suitable polynomial would be $h(x, y) = a + bx + cy$; for (5), $h(x, y) = a + bx + cy + dx^2 + exy + fy^2$; for (6), $h(x, y) = a + bx + cy + dx^2$. If the interpolating function $h(x, y)$ is not so chosen, the formulas remain valid, but derivatives of h will appear.

The same procedure is applicable to functions of any number of independent variables.

ON A LEMMA BY KOLMOGOROFF

By Kai-Lai Chung

Princeton University

The following lemma was proved by Kolmogoroff [1]:

If $e_1, e_2, \cdots, e_n$ are independent events and U an arbitrary event such that ($W(X)$ denoting the probability of X and $W_e(X)$ the conditional probability of X under the hypothesis of e)

$$W_{e_k}(U) \geqq u, \qquad W(e_1 + \cdots + e_n) \geqq u.$$

Then

$$W(U) \geqq \tfrac{1}{9}u^2.$$

This result seems of some interest in itself and may also have practical applications, for it is easily seen that [2] in general if $e_1, e_2, \cdots, e_n$ are arbitrary no information about $W_{e_1+\cdots+e_n}(U)$ can be obtained from that about $W_{e_k}(U)$, $k = 1, \cdots, n$. From this point of view the constant 1/9 is interesting, though it is unimportant in Kolmogoroff's proof of the law of large numbers. Using his original method this constant can easily be improved to 1/8. However, the following method will give a better result. At the same time we shall put it into a more general form.

Let

$$W_{e_k}(U) \geqq \alpha, \qquad \sum_{k=1}^{n} W(e_k) \geqq \beta.$$