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THE FOURTH DEGREE EXPONENTIAL DISTRIBUTION FUNCTION[1]

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We shall derive a recursion formula for the moments of the fourth degree exponential distribution function, state its more characteristic features, and show how the graduation of observed distributions may be accomplished by the method of moments and the method of maximum likelihood. The purpose of the note is to make possible a wider use of this function.

R. A. Fisher [1] introduced the fourth degree exponential function

$$y_t = k \exp \{-(\beta_4 t^4 + \beta_3 t^3 + \beta_2 t^2 + \beta_1 t)\}, \tag{1}$$

where $r_1 \leq t \leq r_2$, $t = (x - m)/\sigma$, m indicates the population mean, σ the population standard deviation, and where the β's are functions of

$$\alpha_n = \int_{r_1}^{r_2} t^n y_t \, dt.$$

A. L. O'Toole in two stimulating papers [2], [3], has studied (1); however his methods and results are unnecessarily complicated. O'Toole requires eight moments to determine parameters similar to the β's. Both Fisher and O'Toole considered the restricted class of (1) with range $(-\infty, \infty)$.

Let

$$u = t^n \exp \{-(\beta_4 t^4 + \beta_3 t^3 + \beta_2 t^2)\}, \; dv = e^{-\beta_1 t} \, dt \tag{2}$$

in

$$\alpha_n = \int_{r_1}^{r_2} t^n y_t \, dt, \quad \text{obtaining} \tag{3}$$

$$4\beta_4 \alpha_{n+3} + 3\beta_3 \alpha_{n+2} + 2\beta_2 \alpha_{n+1} + \beta_1 \alpha_n = n\alpha_{n-1}, \qquad n = 1, 2, 3, \cdots, \tag{4}$$

[1] Presented to the American Mathematical Society and the Institute of Mathematical Statistics, September 4, 1947.