under equivalence [S, P]. A subfield equivalent to a statistic need not itself be a statistic. In an attempt to avoid this difficulty, one may define a *pseudo-statistic* as any subfield equivalent to a statistic. If Lemma 3 remained valid for pseudo-statistics in the sense that a member of C_π is a pseudo-statistic if and only if it is equivalent to $\mathbf{S}_\pi$, this would establish the desired result.

The following example shows that this stronger version of Lemma 3 is not correct. Let $\mathbf{S}_\pi$ be the class of all Lebesgue sets on the real line and $\mathbf{S}_0$ the class of all Lebesgue sets differing only by a set 0 from a set symmetric with respect to the origin. Clearly, $\{x\} \varepsilon \mathbf{S}_0$ for all x so that $\mathbf{S}_0 \varepsilon C_\pi$. Also $\mathbf{S}_0$ is a pseudo-statistic since it is equivalent to the subfield induced by $T(x) = |x|$. But clearly $\mathbf{S}_0$ and $\mathbf{S}_\pi$ are not equivalent.

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A NOTE ON CONFIDENCE SETS FOR RANDOM VARIABLES

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Suppose the chance variables $X_1, \cdots, X_m, Y_1, \cdots, Y_n$ have a joint probability distribution depending on the unknown parameters $\theta_1, \cdots, \theta_k$, but otherwise of known form. We assume that there is a set of sufficient statistics for

$\theta_1, \cdots, \theta_k$, denoted by $T_1(X_1, \cdots, X_m, Y_1, \cdots, Y_n), \cdots,$

$T_r(X_1, \cdots, X_m, Y_1, \cdots, Y_n)$. We shall let X denote the vector $(X_1, \cdots, X_m)$, Y the vector $(Y_1, \cdots, Y_n)$, θ the vector $(\theta_1, \cdots, \theta_k)$, and $T(X, Y)$ the vector $(T_1(X, Y), \cdots, T_r(X, Y))$. $P_\theta(A)$ shall denote the probability of A when the vector of parameters equals θ, and $P_\theta(A \mid B)$ shall denote the conditional probability of A given B when the vector of parameters equals θ.

Given a number α between 0 and 1, if for each vector X we can find a set $S(X)$ in n-dimensional Euclidean space such that $P_\theta(Y \text{ in } S(X)) = \alpha$ identically in θ, then the system of sets $S(X)$ is called a "parameter-free confidence set of level α for the random vector Y."

Since $T(X, Y)$ is a set of sufficient statistics for θ, the joint conditional distribution of Y given that $T(X, Y) = t = (t_1, \cdots, t_r)$ is independent of θ. But then for any given vector t, it is possible to construct a region $S'(t)$ in n-dimensional Euclidean space such that $P_\theta(Y \text{ in } S'(t) \mid T(X, Y) = t) = \alpha$ identically in θ

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