SOME VALIDITY CRITERIA FOR STATISTICAL INFERENCES

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1. Introduction. This paper is concerned with the ways in which existing statistical theories specify the degree of uncertainty of an inference. For the sake of graphic presentation, problems of inference are described in terms of a game between two players—one who makes the inferences and another who questions their validity. Such a model suggests a number of criteria of validity which depend entirely on classical probability calculations. I anticipate that arguments may be advanced for *not* regarding statistical inference as a game; it is hoped that the cogency of such arguments will not prevent the present model from providing some new insight into the problems here considered.

Many, though not all, problems of inference lead to assertions of the type, "The probability that A is true is equal to α," or, "$P(A) = \alpha$." One may ask whether the person making this assertion should be willing to bet that A is true, risking an amount α to win $1 - \alpha$, and should be equally willing to bet that A is false, risking $1 - \alpha$ to win α, against an opponent who has exactly the same information as he and who is allowed to choose either side of the wager. The affirmative answer will not be defended here, but its consequences will be examined.

The game viewpoint is related to, but not identical with, the ideas of von Mises, who has advanced as a postulate "the impossibility of a gambling system" in his definition of probability (see for example [19], p. 15). It has generally been recognized that modern theories of inference, which avoid the assumption of prior distributions of the parameters, should not have the same interpretation as the classical Bayes-Laplace theory based on prior distributions. The present paper attempts to show the sense in which one pays for weakening the classical assumptions by losing the von Mises postulate for the inferences "$P(A) = \alpha$."

Sections 2 to 5 are devoted to the theory of confidence intervals; in Sections 6 to 9 the ideas are generalized to include other statistical problems. The reader is warned not to expect to find any new problems solved in this paper, for at the present stage of development the theory gives at best new ways of looking at existing solutions.

2. A model for studying interval estimation. In the spirit of the introductory section the problem of interval estimation is here studied in terms of a game between two players. The players have equal knowledge about fixed conditions K (for "known") of a random experiment, for example, knowledge that n values are observed from a normal population having unit variance. Unknown conditions of the experiment, for example the value of the population mean μ, are conveniently referred to as the "state of nature" U (for "unknown"). The

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