ABSTRACTS OF PAPERS

(*Abstracts of papers presented at the Stanford Annual Meeting of the Institute, August 23–26, 1960. Additional abstracts will appear in the December 1960 issue.*)

1. Estimating the Infinitesimal Generator of a Finite State Continuous Time Markov Process. ARTHUR ALBERT.

Let $\{Z(t), t > 0\}$ be a separable, continuous time Markov Process with stationary transition probabilities $P_{ij}(t)$, $i, j = 1, 2, \cdots, M$. Under suitable regularity conditions, the matrix of transition probabilities, $P(t)$, can be expressed in the form $P(t) = \exp tQ$, where Q is an $M \times M$ matrix and is called the "infintesimal generator" for the process.

In this paper, a density on the space of sample functions over $[0, t)$ is constructed. This density depends upon Q. If Q is unknown, the maximum likelihood estimate

$$\hat{Q}(k, t) = \|\hat{q}_{ij}(k, t)\|,$$

based upon k independent realizations of the process over $[0, t)$ can be derived. If each state has positive probability of being occupied during $[0, t)$ and if the number of independent observations, k, grows large (t held fixed), then $\hat{q}_{ij}$ is strongly consistent and the joint distribution of the set $\{(k)^{\frac{1}{2}}(\hat{q}_{ij} - q_{ij})\}_{i \neq j}$ (suitably normalized), is asymptotically normal with zero mean and covariance equal to the identity matrix. If k is held fixed (at one, say) and if t grows large, then $\hat{q}_{ij}$ is again strongly consistent and the joint distribution of the set $\{(t)^{\frac{1}{2}}(q_{ij} - q_{ij})\}_{i \neq j}$ (suitably normalized), is asymptotically normal with zero mean and covariance equal to the identity matrix, provided that the process $\{Z(t)\ t > 0\}$ is metrically transitive (but not necessarily stationary) and has no transient states.

The asymptotic variances of the $\hat{q}_{ij}$ are computed in both cases.

2. The Sequential Design of Experiments for Infinitely Many States of Nature. ARTHUR ALBERT. (By title)

In a recent paper (*Ann. Math. Stat.* Vol. 30 (1959), pp. 755–770) Chernoff discussed a problem which he called "The Sequential Design of Experiments" as it applied to the two action (hypothesis testing) case. In that paper, a procedure was exhibited for which the risk was approximately $-c \log c/I(\theta)$, when θ is the true state of nature, $I(\theta)$ is an appropriately defined information number and c, the cost per experimental trial, is small. It was also shown that in order for some other procedure to do significantly better for some value of the parameter, it must do worse by an order of magnitude (as $c \to 0$) at some other value of the parameter. These results were obtained under the assumption that the parameter space is finite. In the present paper, the assumption of finiteness is dispensed with. The procedures proposed here are closely akin to Chernoff's procedure, and analogous (though slightly weaker) optimality properties are derived.

3. Maximal Independent Stochastic Processes. C. B. BELL, University of California, Berkeley. (By title)

R. Pyke (1958) asked: What is the maximum cardinality, M_a, of a family of independent random variables defined on an abstract space Ω of cardinality a? (1) For $a < \aleph_0$, an elementary counting process yields $M_a = [\log_2 a]$. (2) For $a = \aleph_0$, a construction and a result of E. Marczewski (*Colloq. Math.*, 1955) yield $M_{\aleph_0} = \aleph_0$. (3) $M_{\mathfrak{C}} = 2^{\mathfrak{C}}$ follows from a result