ESTIMATION OF ONE OR TWO PARAMETERS OF THE EXPONENTIAL DISTRIBUTION ON THE BASIS OF SUITABLY CHOSEN ORDER STATISTICS

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1. Introduction. Assume that n independent observations are made on a random variable with the exponential distribution function

$$F(x) = 1 - e^{-(x-\alpha)/\sigma} \text{ for } x \geqq \alpha, \tag{1.1}$$

where $\sigma > 0$. We are interested in the estimation of σ and/or α on the basis of k suitably chosen order statistics $x(n_1), x(n_2), \cdots, x(n_k)$, where $n_1, n_2, \cdots, n_k$ are the ranks satisfying the inequalities $1 \leqq n_1 < n_2 < \cdots < n_k \leqq n$.

The asymptotic situation $(n \to \infty)$ has been considered by Ogawa (1960) and Kulldorff (1962), while the small sample situation has been dealt with by Harter (1961) and Siddiqui (1963) for $k = 1$, and by Ukita (1955), Harter (1961), Sarhan, Greenberg and Ogawa (1963) and Siddiqui (1963) for $k = 2$. This paper deals with the small sample situation for any value of k. We shall derive the best linear unbiased estimates (BLUE's) of

(i) σ when α is known (Section 2),

(ii) α when σ is known (Section 3),

(iii) σ and α when both parameters are unknown (Section 4).

We shall also present some theoretical and numerical results concerning the optimum choice of the ranks.

In deriving the various BLUE's, we shall make use of the following results due to Gumbel (1937) and Sarhan (1954):

$$E[x(n_i)] = \alpha + \sigma \sum_{j=1}^{n_i} (n - j + 1)^{-1}, \tag{1.2}$$

$$V[x(n_i)] = \text{Cov}\,[x(n_i), x(n_{i'})] = \sigma^2 \sum_{j=1}^{n_i} (n - j + 1)^{-2} \qquad (i < i'). \tag{1.3}$$

It will be convenient to employ the notation

$$\delta_{ri} = \sum_{j=n_{i-1}}^{n_i - 1} (n - j)^{-r} \quad (r = 1, 2; i = 1, 2, \cdots, k), \tag{1.4}$$

where $n_0 = 0$. Whenever the quantities δ_{10}/δ_{20} and $\delta_{1,k+1}/\delta_{2,k+1}$ appear in the sequel, it will be understood that they are equal to zero.

We shall also use the following well-known theorem, which is usually associated with the names of Gauss and Markoff.

THEOREM 1. *Let X be a $k \times 1$ vector of observations, let θ be a $s \times 1$ vector of*

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