BAYESIAN BIO-ASSAY[1]

By Charles H. Kraft and Constance van Eeden

University of Minnesota

1. Introduction. The basic bio-assay problem can be stated as follows. Let F be a distribution function and let $Y = (Y_1, \cdots, Y_k)$ be a set of k independent random variables, each of which is binomial with parameters $(n_i, F(t_i))$. The numbers $t_1, \cdots, t_k$ are called dosage levels. The experimenter knows the n_i and the t_i, can observe Y and wants to make some inference about F.

The approach we discuss here is Bayesian, that is F is random and the distribution of Y just described is taken to be the conditional, given F, distribution of Y.

In the following sections a characterization of the class of all (a priori) distributions for F is given, the corresponding Bayes' estimates for a class of loss functions are found and the results of LeCam [5] are applied to show completeness of the closure of this class of estimates for a certain topology. A special case is given for which the estimates are explicitly computed.

2. A priori distributions for F. The set of distribution functions, F, is often taken to be a parametric family. In such a case an a priori distribution can be given for the parameters. It is, however, possible to be completely general. Namely let $D = \{d_i\}$ be a countable dense subset of the real line, and let a sequence of probability laws $\mathfrak{L}[F(d_1)]$, $\mathfrak{L}[F(d_2) \mid F(d_1)]$, $\cdots$, $\mathfrak{L}[F(d_n) \mid F(d_1), \cdots, F(d_{n-1})]$, $\cdots$ be given such that $P(F$, on D, is a distribution function$) = 1$. Then, defining for $x_0 \not\in D$, $F(x_0) = \lim_{x \to x_0^+, x \in D} F(x)$, defines P for F with $P(F$ is a distribution function$) = 1$. It is clear that this construction yields a separable process and, also, that any process that produces, with probability one, distribution functions will have such a separable representation.

The above given construction specifies an a priori distribution for F by giving the joint distribution of the ordinates of F at certain fixed abcissa. Another way to specify a distribution for F is to give, consistently, the joint distribution of the percentiles of F.

3. The loss functions. The loss functions we wish to consider are the following. Let $W(x)$ be an arbitrary (fixed) distribution function. If G, a non-decreasing bounded between 0 and 1 function on the real line, is the statistician's decision and F the distribution determining the distribution of Y, then the loss $L(F, G) = \int (F - G)^2 \, dW$. For this loss the Bayes' estimate is the conditional, given Y, expectation of the process. The proof follows immediately from the usual pointwise (in Y) construction of Bayes' procedures.

Received 19 March 1963; revised 10 August 1963.

[1] This research was supported in part by the Office of Naval Research under Grant Number Nonr 710(31), by the National Science Foundation under Grant Number G-19126 and by the National Institutes of Health under Grant Number 2G-43(C7+8).