ON BIVARIATE RANDOM VARIABLES WHERE THE QUOTIENT OF THEIR COORDINATES FOLLOWS SOME KNOWN DISTRIBUTION

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1. Introduction. Let X_1, X_2 be a pair of independent random variables, symmetrical about the origin, having the same distribution function $F(x)$, and let the quotient

$$Z = X_1 : X_2 \tag{1.1}$$

follow the Cauchy law. It is known that $F(x)$ may be normal with zero mean. A number of authors investigated whether the normal distribution can be characterized by this property. Mauldon [9], Laha [4] and Steck [11] showed this supposition to be false; there exist distribution functions $F(x)$ differing from the normal, where the quotient (1.1) follows the Cauchy law. Denote by C the set of distribution functions $F(x)$ having the above-mentioned property; Laha [5], [6] and Kotlarski [2] undertook a study of characterizing the set C. Kotlarski [2] characterized the set C by the properties of the Mellin transform $h(s)$ of $F(x)$ given by the formula

$$h(s) = \int_{-\infty}^{+\infty} |x|^s \, dF(x). \tag{1.2}$$

On this subject see also [7] p. 324 and [8] p. 178.

In this paper we shall consider a bivariate random variable (X, Y) having distribution $F(x, y)$, where the coordinates (not necessarily independent) have identical marginal distributions $F(x, \infty) = F(\infty, x)$, $(-\infty < x < +\infty)$ and the quotient

$$Z = X : Y \tag{1.3}$$

follows the Cauchy law (Section 3). The set $\mathfrak{X}$ of such distribution functions $F(x, y)$ will be described by using their two-dimensional Mellin transforms (see Section 2).

With the same method we describe in Section 4, the set $\mathfrak{Y}$ of distribution functions $F(x, y)$ of bivariate random variables (X, Y) having positive coordinates (not necessarily independent or identically distributed), where the quotient (1.3) follows Snedecor's law. On this subject see also [3], [9].

Further in a similar way may be described the set of distribution functions $F(x, y)$ of bivariate random variables (X, Y) where X has a symmetrical distribution about the origin and Y takes positive values only, X and Y not necessarily being independent, where the quotient (1.3) follows Student's law.

2. The Mellin transforms of bivariate random variables whose coordinates take only positive values. We define the Mellin transform of a bivariate random

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