BOOK REVIEWS

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ULF GRENANDER, *Probabilities on Algebraic Structures.* Wiley, New York and London, 1963, $12.00 and £4/5/10. 218 pp.

Review by G. KALLIANPUR

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In recent years, particularly within the last two decades, a great deal of work has been done in developing the theory of probability measures on topological groups and semi-groups, linear topological vector spaces (such as Banach spaces and the space of Schwartz distributions) and topological algebras. From a theoretical point of view this work can be regarded as an attempt to extend to these more general topological-algebraic structures the concepts and techniques that have proved so successful in studying Euclidean probability spaces, as well as to discover new ones. The impetus for such a theory has come both from its mathematical appeal and from a variety of (apparently unrelated) practical problems from communication engineering, physics and, not least, from statistics. The most significant contributions in this rapidly growing field are to be found widely scattered in mathematical journals and symposia and hitherto no unified account of these researches has been available to the interested student of probability and statistics. The present book is the first, necessarily tentative, effort in this direction, no doubt to be followed in the years to come by more ambitious and definitive studies.

The first chapter opens with a brief survey of the methods and results of the "classical" theory of real random variables, and proceeds to give a motivation for its extension by a discussion of several practical problems, followed by a short historical background. The author is anxious to convince the reader at the very start that, despite the highly abstract nature of the theory, its aim is not to generalize for the sake of generalization.

The subject proper is introduced in Chapter 2, where one begins by considering a space which is a topological semi-group (associative but not necessarily commutative) with probability measures defined on the Borel field generated by all open sets. There is now enough algebraic structure to define the convolution of probability measures and to launch the study of properties of products (or sums if the semi-group operation is thought of as addition) of independent random elements. The same chapter gets down to a more detailed study of compact semi-groups. A whole group of problems now unfolds itself: the limiting behavior of products of independent random elements, in particular, the law of