

THE RELATIONSHIP BETWEEN SUFFICIENCY AND INVARIANCE WITH APPLICATIONS IN SEQUENTIAL ANALYSIS

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PREFACE. The main result in this paper relating sufficiency and invariance was originally found by Charles Stein before 1950 but was not published and not widely known. It has since been rediscovered independently by Burkholder in 1958 (reported in [7]), by Hall in 1959 ([18], [19]), and by Ghosh in 1960 [16]; the best theorems of this kind have since been developed by Wijsman. This result is closely related to a theorem of D. R. Cox [9], published in 1952 and widely used in sequential analysis (e.g., [14], [17], [22], [23]) though Cox made no explicit use of invariance concepts. The result, together with extensions (due to Wijsman), related results on transitivity (due to Ghosh), and sequential applications (due to Hall and Ghosh), is now finally published as a joint contribution, with the permission of Stein and Burkholder.

This paper is presented in two parts. Part I, largely written by Hall and Ghosh, discusses the implications of the main result and sketches a proof. It also discusses a result in transitivity and the application of it and the main result to sequential analysis. Several normal theory examples and a sequential rank test are treated in some detail. Part II, largely written by Wijsman, presents the general theory in the subfield mode, including related results on conditional independence and transitivity, and additional examples.

The authors wish to thank H. K. Nandi for the research guidance given to one of them, D. L. Burkholder for helpful discussions, and E. L. Lehmann, J. L. Hodges, Jr. and W. Kruskal for making this joint endeavor possible.

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Received 29 October 1964.

¹ Research was supported in part by the Air Force Office of Scientific Research, the Office of Naval Research, and the National Institutes of Health under Grant GM-10397. Reproduction is permitted for any purposes of the United States Government.

² Research was supported by the National Science Foundation under Grants G-11,382 and G-21,507.

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