ASYMPTOTIC OPTIMUM QUANTILES FOR THE ESTIMATION OF THE PARAMETERS OF THE NEGATIVE EXPONENTIAL DISTRIBUTION

By A. K. Md. Ehsanes Saleh and Mir M. Ali

University of Western Ontario

1. Introduction. Assume that we are sampling from the negative exponential distribution

$$F(x) = 1 - e^{-(x-\mu)/\sigma}, \qquad x \geqq \mu;\ \sigma > 0, \tag{1.1}$$

where μ and σ are respectively the location and the scale parameters. Denote the observations of an ordered random sample of size n by $x_{(1)} < x_{(2)} < \cdots < x_{(n)}$. The integer i denotes the rank of the order statistic $x_{(i)}$.

The problem of estimation of μ and σ simultaneously (or one of them when the other is known) based upon a fraction of the sample, say, the observations, $x_{(n_1)} < \cdots < x_{(n_k)}$ arises frequently in practice. The integer k denotes the number of observations for the estimation. In such estimation problem linear estimates, $c_1x_{(n_1)} + c_2x_{(n_2)} + \cdots + c_kx_{(n_k)}$, are useful. The quantities $c_1, c_2, \cdots, c_k$ are the coefficients of the linear function. When both μ and σ are unknown and to be estimated, we obtain two linear functions differing only in coefficients but the sample observations remain the same. The observations that are used to form the estimate will be the relevant sample for the estimation. In such estimation problem, ranks n_i $(i = 1, \cdots, k)$ of the observations in the relevant sample form a subset $R_k = \{n_1, n_2, \cdots, n_k\}$ of the set $I_n = \{1, 2, \cdots, n\}$. Therefore, in estimating the parameters based on k $(\leqq n)$ observations we have at our disposal $\binom{n}{k}$ subset of the ranks to determine the relevant sample. Some subsets $R_k^0 = \{n_1^0, \cdots, n_k^0\}$ are preferable to others if estimates based on the corresponding order statistics $x_{(n_1^0)}, x_{(n_2^0)}, \cdots, x_{(n_k^0)}$ possess minimum variance property among all other $\binom{n}{k}$ estimates. If such a set R_k^0 exists, the corresponding set of order statistics will be called "optimum set." Therefore, the problem arises as to how the "optimum set" of order statistics be determined for fixed values of n and k such that the linear estimates of μ and σ based on them have optimum variance property.

The problem has been studied numerically by several authors. The small sample situation has been studied by Harter (1961), Kulldorff (1963), Sarhan, Greenberg and Ogawa (1963), Siddiqui (1963). The asymptotic situation $(n \rightarrow \infty)$ for the estimation of σ when $\mu = 0$ has been considered by Kulldorff (1962), Ogawa (1950) and Sarhan, Greenberg and Ogawa (1963). In this paper, we present the asymptotic situation $(n \rightarrow \infty)$ and obtain analytically a system of equations which determine the k $(\leqq n)$ optimum spacings of the optimum set

Received 9 January 1964; revised 19 July 1965.