BOUNDED LENGTH CONFIDENCE INTERVALS FOR THE p-POINT OF A DISTRIBUTION FUNCTION, II[1]

By R. H. Farrell

Cornell University

1. Introduction. Let $0 < p < 1$. A number $\gamma_{p,F}$ is a p-point of the distribution function F if $F(\gamma_{p,F}) \geqq p$ while $F(\gamma_{p,F}-) \leqq p$. Given $L > 0$ and $0 < \alpha < 1$, a (sequential) confidence interval procedure is a L-α bounded length confidence interval procedure if when sampling stops an interval of length not exceeding L is given which covers $\gamma_{p,F}$ with probability at least $1 - \alpha$.

Throughout this paper we consider only those decision procedures that are based on independently and identically distributed random variables. In the sequel we make this completely precise. We present a negative result (see the statement of the theorem below). It will be convenient to say simply "there does not exist a procedure that works for all $F \varepsilon \mathbf{F}$." This means a set $\mathbf{F}$ of distribution functions is specified and that the common distribution function F of the random variables is in $\mathbf{F}$. The problem is to obtain a confidence interval of length $\leqq L$ for the p-point $\gamma_{p,F}$ of the common distribution function F. The experimenter is allowed to construct his procedure using the information $F \varepsilon \mathbf{F}$. We show that there cannot exist confidence interval procedures of a specified type giving an interval of length $\leqq L$ yet covering $\gamma_{p,F}$ with probability $\geqq 1 - \alpha$ for all $F \varepsilon \mathbf{F}$. Thus, for example, it is clear that no sequential confidence interval procedure can work for all F satisfying $F(\gamma_{p,F} - L-) = F(\gamma_{p,F} + L)$.

In Section 2 of the first of this series of papers, Farrell [3], we define a measure of flatness by

$$(1.1) \quad \epsilon_F = \sup_{0<\rho<1} \min\,(F(\gamma_{p,F} + \rho L) - p,\ p - F(\gamma_{p,F} + (\rho - 1)L)).$$

We shall be interested in confidence interval procedures that may be applied to observations on $F \varepsilon \mathbf{F}$, where if $F \varepsilon \mathbf{F}$ then $\epsilon_F > 0$.

We are interested in choices of $\mathbf{F}$ for which fixed sample size procedures will fail to be L-α bounded length confidence interval procedures. In case $F \varepsilon \mathbf{F}$ implies that F has a unimodal density function, Weiss [5] has shown the existence of two-stage L-α bounded length confidence interval procedures. In the general case of $F \varepsilon \mathbf{F}$ if and only if $\epsilon_F > 0$ we show in [4] the construction of a sequential L-α bounded length confidence interval procedure with certain optimality properties. Examples of sequential procedures have also been constructed by J. Kiefer and L. Weiss but these examples have not been published.

The present paper gives a nonexistence result.

Received 3 September 1965.

[1] Research sponsored in part by the Office of Naval Research under Contract Nonr 401(50) with Cornell University. Part of this work appears in the author's Ph.D. dissertation, University of Illinois.