COMPARISON OF COMBINED ESTIMATORS IN BALANCED INCOMPLETE BLOCKS

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0. Introduction. In the analysis of Balanced Incomplete Block designs (BIB), there arise two independent estimates of treatment differences conventionally referred to as the intra-block and the inter-block estimates. Yates [4] showed that an inter-block analysis can be made for the BIB assuming that the block effects are random. He also devised a method for combining this additional information with the customary intra-block information so as to estimate the treatment differences with greater precision than if the intra-block information had been used alone. An alternative combined estimate has been suggested by Graybill and Weeks [1] and shown to be unbiased. In this paper we compare the two estimates and answer the question raised by Graybill and Weeks [1] as to which estimate is better in the sense of smaller variance.

1. Notations and assumptions. We refer the reader for a fuller discussion of the model to Graybill and Weeks [1], and state only those assumptions which pertain to the problem considered in this study.

(i) The $(t-1)\times 1$ vector $U=(u_i)$ is normally distributed with mean $T=(t_i)$ and covariance matrix $(k/\lambda t)\sigma^2 I$, so that u_i (referred to as the intra-block estimate) is unbiased for t_i and has variance $(k/\lambda t)\sigma^2$.

(ii) The $(t-1)\times 1$ vector $X=(x_i)$ is normally distributed with mean $T=(t_i)$ and covariance matrix $\{k(\sigma^2+k\sigma_\beta^2)/(r-\lambda)\}I$, so that x_i (referred to as the inter-block estimate) is unbiased for t_i and has variance $k(\sigma^2+k\sigma_\beta^2)/(r-\lambda)$.

(iii) s_1^2/σ^2 has a chi-square distribution with $f=(bk-b-t+1)$ degrees of freedom.

(iv) $s_2^2/(\sigma^2+k\sigma_\beta^2)$ has a chi-square distribution with $(b-t)$ degrees of freedom, where $b>t$.

(v) $u_1, u_2, \cdots, u_{t-1}, x_1, x_2, \cdots, x_{t-1}, s_1^2, s_2^2$ are all mutually independent.

The following notations will be used in the paper.

(a) $E_x(\cdot)$ denotes the expectation of $(\cdot)$ in the space of x.

(b) $E_{x|y,z}(\cdot)$ denotes the expectation of $(\cdot)$ over fixed values of y and z.

(c) $E_{s_1^2,s_2^2,z_i}(\cdot)$ will be referred to by $E'(\cdot)$.

(d) $V(\cdot)$ denotes the variance of $(\cdot)$.

(e) Yates' estimate is denoted by $\bar{T}_i$, and Graybill and Weeks' estimate by $\hat{T}_i$.

(f) $P(x>a)$ denotes the probability that $x>a$.

2. Statement of the problem. The object of the present study is to compare

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