A. V. Skorokhod, *Studies in the Theory of Random Processes*, Addison-Wesley, Reading, 1965. $12.50. viii + 199 pp.

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This is an excellent little book dealing with several areas in the theory of Markov processes. The volume under review is a translation of a Russian monograph which appeared originally in 1961. Aside from an introductory first chapter, the book consists roughly of two parts. The first (Chapters 2–5) concerns itself with a class of Markov processes that can be realized as the solutions of stochastic integral equations—the class is broad enough to include the familiar diffusion processes and also processes with independent increments. The second part (Chapters 6 and 7) is devoted to various limit theorems connected with the convergence of a sequence of Markov chains to a Markov process with continuous time.

Chapter Two presents the theory of stochastic integrals along the lines developed by K. Ito. In particular Ito's formula for "stochastic differentials" is included as is a discussion of the continuity of a stochastic integral as a function of its upper limit of integration. In Chapter Three the author uses stochastic integrals to construct a wide class of Markov processes as solutions of certain stochastic integral equations. Again the author follows the development of Ito. The author works consistently in Euclidean m-space R^m. Various existence and uniqueness theorems for stochastic integral equations are presented. In Chapters 3 and 4 the author investigates conditions under which the probability measures of two Markov processes are equivalent and computes the Radon-Nikodym derivative in various situations. Special attention is given to processes with stationary independent increments in R^m and to one dimensional diffusion processes.

Chapters 6 and 7 deal with the type of limit theorems that are often called "invariance principles" in the Western literature. Much of the material in these chapters is due to the author. In Chapter 6 one finds the J-topology and conditions for the convergence of the distributions of J-continuous functionals. Conditions are given for the convergence of the finite dimensional distributions of approximating chains to the finite dimensional distributions of the solution of a given stochastic integral equation. In Chapter 7 the author treats various problems in the theory of sums of independent random variables. His treatment is based on the following result: Let $\xi_1, \xi_2, \cdots, \xi_n$ be independent random variables with $E(\xi_i) = 0$ and $E(\xi_i^2) = \sigma_i^2 < \infty$ and let $w(t)$ be a Brownian motion process with $w(0) = 0$. Then there exist nonnegative independent random variables $\tau_1, \tau_2, \cdots, \tau_n$ such that if $T_k = \tau_1 + \cdots + \tau_k$, $1 \leqq k \leqq n$, then $w(T_1)$, $w(T_2) - w(T_1), \cdots, w(T_n) - w(T_{n-1})$ have the same joint distribution as $\xi_1, \xi_2, \cdots, \xi_n$ and $E(\tau_i) = \sigma_i$, $1 \leqq i \leqq n$.

The translation by Scripta Technica is, at best, uneven. The price of $12.50 for this slim volume of 199 pages seems extremely high to the reviewer. In conclusion this is a very useful work that, aside from its prohibitive price, can be recommended to all serious students of stochastic processes.